Applying Physics Makes Auditory Sense

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A New Paradigm in Hearing

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and

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We, the authors, would both like to take this opportunity to express our deepest gratitude to the women who encouraged us to follow our inspiration, our wives

_Elly Heerens_ and _Manou de Ru_

together with both of our families, they offered their unconditional support and enduring patience during the many years that it took to finally reach publication of our work.

Not the end, but merely a beginning!
Nietzsche’s Ode to Physics

--- Und dazu müssen wir die besten Lerner und Entdecker alles Gesetzlichen und Notwendigen in der Welt werden: wir müssen Physiker sein, um, in jenem Sinne, Schöpfer sein zu können, — während bisher alle Wertschätzungen und Ideale auf Unkenntnisse der Physik oder im Widerspruch mit ihr aufgebaut waren. Und darum: Hoch die Physik! Und höher noch das, was uns zu ihr zwingt, — unsre Redlichkeit! ---

Friedrich Wilhelm Nietzsche
1882 ‘Die fröhliche Wissenschaft’ Buch IV ‘Sanctus Januarius’

--- And to that end we must become the best learners and discoverers of everything that is lawful and necessary in the world: we must become physicists in order to be able to be creators in this sense, — while hitherto all valuations and ideals have been based on ignorance of physics or were constructed so as to contradict it. Therefore: long live physics! And even more so that which compels us to turn to physics, — our honesty! ---

Translated by: Walter Arnold Kaufmann
1974 ‘The Gay Science’

Prologue

In his time Daniel Bernoulli (1700 – 1782) was widely known as an accomplished physicist, mathematician and physician. Among his many scientific achievements the most important discovery by far was a phenomenon that to this day is known as:

“Bernoulli’s Law”.

Bernoulli noticed that the expression that describes the energy content of a moving fluid in the ‘Law of Conservation of Energy’ – an important guiding principle in physics that expresses that the total energy, the sum of kinetic and potential energy, in a mechanical system always remains constant – requires an extra term related to the internal pressure of that fluid.

Subsequently, he realized that the kinetic energy per volume $E_{\text{kin}}/V$ inside a fluid that streams homogeneously through a tube with velocity $v$, would increase with an amount $\rho v^2$ if the velocity is altered from zero to $v$, and the internal fluid pressure simultaneously decreases by the amount of $\Delta p$. The factor $\rho$ in this equation stands for the fluid’s density or mass per volume. Naturally, there are some constraints: i.e. the fluid must be incompressible, non-viscous, and the flow must be non-turbulent. Bernoulli’s law indeed offers highly precise experimental results as long as these conditions are met to a reasonable degree.

Bernoulli reported that this principle could also be used to measure the flow of blood through the veins. We cannot be sure, we suspect however, that Bernoulli himself may have attempted this procedure on his own patients. The records show that at this time throughout Europe physicians started to use hollow needles, which they inserted into their patient’s veins in order to measure the blood flow presumably based on their knowledge of the Bernoulli effect. Thus, we may conclude that it had become widely known that one could detect changes in the flow of a fluid that passes through a thin tube by measuring the change in pressure on that tube’s wall.

In addition to his impressive mathematical work Bernoulli also left us with an in-depth study with regard to the vibrations of strings in musical instruments.

Only a hundred years later, Hermann von Helmholtz (1821 – 1894) another renowned physician and physicist was able to study both the brilliant work of anatomist Alfonso Corti (1822 – 1876) with regard to the anatomy of the inner ear, and the magnificent mathematical work regarding periodic functions and composite vibration phenomena of Jean Baptiste Joseph Fourier (1768 – 1830).

The knowledge that Von Helmholtz thus obtained inspired him to undertake a first serious attempt to describe and explain the functioning of the mammalian inner ear – presenting the scientific world with the first widely accepted hearing paradigm.
Over the course of time this paradigm was replaced by new concepts, such as the traveling wave paradigm by Nobel prize laureate Georg von Békésy (1899 – 1972). Von Békésy’s paradigm – albeit substantially modified and adapted – still endures.

I am convinced that history would have been rewritten, if Bernoulli – a versatile genius – had similar access to the work of these fellow scientists, as I do now. Detailed knowledge of the anatomy of the inner ear including knowledge of the flow of perilymph fluid in the tube-like scalae within the cochlea, understanding the mathematics of periodic functions including vibrations, together with the knowledge that Bernoulli had himself obtained through his own discoveries, would inevitably have inspired Bernoulli to formulate a completely different explanation of the functioning of the mammalian auditory sense.

Bernoulli’s conclusions would most likely have inspired him to formulate yet another hearing paradigm. Inescapably, this new concept would also have altered the currently accepted model of the auditory sense that is based on Von Békésy’s model.

The Seebeck-Ohm-Helmholtz controversy – an ongoing debate centered on the question whether the pitch that we hear is actually built up from the contributing frequencies in a tone complex – would probably not even have come into existence.

The different schools of thought that developed with regard to the explanation of this perception phenomenon would not have come into being either. Neither the phenomenon of the missing fundamental – a sound that is nevertheless heard by us – nor the hypothesis of the traveling wave propagating over the basilar membrane – as proposed by Georg von Békésy – would have been explained as they were, and still are to this day.

*Ironically, our initiative to publish this booklet would not have been necessary either.*

By combining all the physics and mathematics of the subject matter Bernoulli would probably have noticed that the hydro-dynamical effect that carries his name also plays a crucial role inside the cochlea, this would more than likely have lead him to the conclusion that:

*The incoming sound signal is transformed into the sound energy signal inside the cochlea. It is this signal that evokes both the mechanical vibrations in the basilar membrane and the corresponding electrical stimuli in the organ of Corti, stimuli that are subsequently sent to the brain in a frequency selective manner.*
Unfortunately, Bernoulli did not have this knowledge at his disposal and therefore he could not complete this exercise. Now, more than 150 years after von Helmholtz’s effort, we have successfully completed a full re-assessment of the auditory sense.

We bring to the table my many years of experience as an all-round physicist and mathematician, and the expertise of my co-author J. Alexander de Ru, a physician and researcher specialized in otolaryngology, both of us blessed with a strong tendency towards thinking outside of the box.

In conclusion, our discoveries and our subsequent new model of the auditory sense are solidly based on the re-interpreted experimental data of Wever and Lawrence; and functionally equate to the mathematical statement that our auditory sense

\[ \text{differentiates and squares the incoming sound stimulus.} \]

Based on our insights derived from literature we arrive at two more basic principles that form the cornerstones of our model: namely, the fact that the attenuation of the eardrum and the ossicular chain are at the root of the extremely large dynamic range of our auditory sense, and the fact that the bone conduction phenomenon is actually the result of the push-pull movement of the perilymph fluid instead of the presumed deformation of the bony structures.

\[ \text{These three main premises form the basis for this} \]
\[ \text{new paradigm of the human auditory sense.} \]

Despite the many years that we have unsuccessfully attempted to achieve publication, thwarted by inexplicable problems during review – familiar phenomena described by Thomas Kuhn as part of the inevitable process during a scientific revolution ultimately leading to a paradigm shift – my co-author and I, both remain committed to the possibility of furthering our respective fields of science.

We are therefore convinced that we are obliged to bring all of these insights to your notice in the form of this booklet.

Willem Christiaan Heerens

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1. Introduction

1.1 Objections against the traveling wave hypothesis

Current models of the cochlea are based on Von Békésy’s [1] hypothesis that a traveling wave propagates over the basilar membrane in the cochlea. In Von Békésy’s theory, sound pressure variations in front of the eardrum – transferred by the ossicular chain – evoke pressure waves inside the cochlea. These subsequently set the basilar membrane into a traveling wave motion running from the base, nearby the area of the oval window, to the apex or helicotrema. It is generally assumed that this traveling wave transfer mechanism generates a maximum deflection at a specific location on the basilar membrane and then extinguishes rapidly thereafter in the direction of the helicotrema. This deflection subsequently evokes an electric signal in the organ of Corti, which is transferred to the auditory cortex via the auditory nerve.

However, a traveling wave carrying vibration energy to generate the expected deflections on the basilar membrane for any of the frequencies in the audible spectrum is not consistent with the fundamental laws of physics.

The viscosity coefficient of perilymph fluid is approximately three times that of water, and perilymph is therefore still considered to be a low viscous fluid. The velocity of sound wave propagation inside this liquid is comparable with the speed of sound in water, which is approximately 1500 m/s. If this is combined with the travelling wave equation $v = f \times \lambda$, where $v$ is the propagation velocity of the wave in m/s, $f$ the frequency in Hz and $\lambda$ the wavelength in m, it is clear that even for the highest audible frequency of 20 kHz, its 75 mm wavelength does not even fit the length of the basilar membrane, which measures 35 mm.

Furthermore, we need to establish that a sound wave will take the line of least resistance, in this case, via the fluid and not via the membrane. This also applies to the vibration energy.

In 1954, this issue was already the subject of a general and fundamental dispute as reported by Wever, Lawrence and Von Békésy [2]. In current cochlear models this cardinal question, answering how and where sound wave energy propagates in the cochlea, still remains unsolved.

Besides, cochlear experts have erroneously suggested that the ‘wave’, which is observed to be running along the basilar membrane and is therefore, commonly interpreted as a traveling wave, can be compared to the ‘ripples’ in a pond.
However, a simple scale-up of such vast proportions, comparing the dimensions of the tiny cochlea to those of a pond, is impossible for more than one reason. Capillary waves [3], or rather interfacial waves, are only possible on the interface between two immiscible fluids, or between a fluid and a gas, provided that two cardinal conditions are met.

The first constraint: a solid partition may not disturb the interface between both media. Not even when that partition is extremely thin in relation to the amplitudes of the movements in that interface, as capillary waves contain both transversal and lateral interface movements. The presence of the relatively solid and substantial basilar membrane as a partition in the cochlea simply prevents the existence of the necessary lateral movements.

The second required constraint for capillary waves is that the fluids on either side of the interface must possess significantly different densities. However, the difference in density between both fluids in the cochlea, perilymph and endolymph, is only rudimentary.

As a consequence, the commonly hypothesized conditions for capillary waves – ‘the ripples in a pond’ – do not exist within the cochlea.

Aside from the aforementioned constraints there is also one fundamental, mechanical constraint that makes it impossible for both forward and backward traveling waves to run freely over the basilar membrane. It has been established by the experimental work of Von Békésy [1], which was later confirmed by numerous other scientific investigators, that the mechanical properties of the basilar membrane are very peculiar indeed.

Determined by the combination of local width, stiffness and stress it possesses a logarithmically distributed resonance topography. Low resonance frequencies of this membrane are to be found near the helicotrema and high resonance frequencies near the round window [4]. Starting at approximately 20 Hz this distribution spans almost three decades in frequency up to 20 kHz over a human basilar membrane length of approximately 35 mm. The over a short distance rapidly changing mechanical properties in the cochlea clearly show that the signal transfer mechanism of the cochlea cannot be regarded as a well-tuned transmission line [3].

A transmission line can only adequately transmit vibration energy of a specific frequency when everywhere along this line the characteristic impedance is properly adapted to the frequency of that signal [5]. A vibration signal with frequency $f_s$ will otherwise be extremely dampened and will even extinguish outside of the local area with resonance frequency $f_r$, where $f_r = f_s$. 
1.2 Objections against the cochlear amplifier hypothesis

The extensive dynamic range of approximately 90 dB in the auditory sense has puzzled scientists for many decades. Nowadays, experts are convinced that this could not be achieved without an amplifier [6-8]. These last three decades the usually hypothesized solution presumes that the outer hair cells in the cochlea do not play a direct role in the transfer of electrical signals to the auditory cortex, but that they amplify the stimulus by their motility [9]. Although the discussion continues, the aforementioned has led to the hypothesis that the outer hair cells possess such a motility that they are capable of enlarging the deflections of the basilar membrane, and in this way function as a cochlear amplifier.

Furthermore, in cooperative action, they are thought to generate movements in the basilar membrane, which in turn evoke backwards-traveling waves towards the oval window. At the oval window, these supposedly backwards-traveling ‘sound waves’ are transferred via the ossicular chain to the eardrum, where they can then be detected as the commonly observed oto-acoustic emission (OAE) signals [10,11].

Besides, it is also hypothesized that the signal transfer inside the cochlea consists of electrical currents generated by the deformation of the inner hair cells. Shearing forces between the edge of the moving basilar membrane and the somewhat tilted tectorial membrane inside the scala media were presumed to generate this deformation.

The major objection against these hair cell hypotheses is that a physically impossible task is expected of these outer hair cells. Each one of them has to generate forces to accelerate an amount of mass many times larger than its own. In our opinion, even if all outer hair cells are cooperating in a coherent way, the combined forces will still be too weak to create a detectable signal.

A contribution by Ren and Gillespie [12] to this discussion confirms our opinion. They concluded their theoretical and experimental research with the following statement:

--- However, how the cochlea employs outer hair cell generated forces to amplify cochlear-partition vibration remains unanswered. In spite of three decades of intensive studies, the expected power gain of the cochlear amplifier has not been demonstrated experimentally. The extremely restricted longitudinal pattern and location of the cochlear nonlinearity at the peak-response place are both inconsistent with predictions based on available theories. ---
Therefore, it would be impossible to evoke a backwards-traveling wave, as outer hair cells would not only have to generate local basilar membrane movements, but would also have to realize relatively large displacements inside the cochlea of both the endolymph and perilymph fluids in a successive, systematic and area restricted action. This in order to generate the expected traveling waves.

And indeed experiments of De Boer et al. [13] confirm the findings of Ren et al. [14] and He et al. [15]: Backwards traveling waves that can be held responsible for distortion product OAE’s in the cochlea are not found. Only forward traveling waves related to the evoked OAE’s are observed in their experiments.

### 1.3 Objections against the existing bone conduction signal transfer hypothesis

The common model for the transfer of a bone conducted signal is based on the vibration of the temporal bone. However, both the material and the construction of this bony envelope are extremely rigid, while the cochlear shape of the cavity excludes the possibility that parts of this cavity can function as the hypothesized resonators. Therefore, sound stimuli cannot be transferred in such a manner. Certainly not the weak vibration signals with pressure variations of approximately 20 mPa, equal to a pressure variation of $0.2 \times 10^{-6}$ atmosphere that are generated by a normal sound stimulus of 60 dB SPL. Hence ‘bone conduction’ must have a cause other than the deformation of the rigid petrous bone that forms the cochlear bony envelope.

### 1.4 Objections against the existing description of inner ear hydrodynamic behavior

In his description of the functioning of the cochlea Von Békésy [1] has stated that the Reissner membrane, which forms the separation between the perilymph filled scala vestibuli and endolymph filled scala media, hardly possesses any internal tension and is so thin that it does not form an obstacle for pressure induced flows of either perilymph or endolymph. He concluded that these two facts allow him to ignore the existence of the Reissner membrane when observing the hydrodynamic behavior of the surrounding fluids. As a consequence, in his opinion, the three inner ear ducts, the scala tympani, scala vestibuli, and scala media are considered to be a combination of scala tympani and scala vestibuli only, interconnected at the helicotrema and separated by the basilar membrane.

However, the Reissner membrane remains a solid thin partition. The movements of this membrane that are a result of the perpendicular movements of the fluids on
either side of its surface will not create severe problems, however, for movements in lateral directions this membrane would have to be infinitely elastic and flexible. Naturally, the Reissner membrane cannot meet these requirements and so it remains an obstacle for the hydrodynamic flow conditions on either side. The hydrodynamic behavior of the perilymph in the scala vestibuli and the scala tympani, and the hydrodynamic behavior of the endolymph in the scala media, are different entities and should consequently be regarded as such.

![Diagram](image)

**Fig. 1.** Acoustic energy flow by two traveling waves explained by Lighthill

Therefore, ignoring either the influence of the Reissner membrane or the existence of the scala media is not permitted. It is precisely this misinterpretation that has led cochlear experts to accept the attempt by Lighthill [5] to theoretically explain the hydrodynamic energy flow inside the cochlea. Aided by the suggestions of leading experts in cochlear research, Lighthill illustrates how, in his opinion, acoustic energy by both a fast and a slow wave might be transported inside the two adjacent tubes that are separated by a flexible partition. His attempt was based on an impressive number of calculations related to various cochlear models, consistently maintaining the presumption of Von Békésy that a traveling wave carries the sound energy to a specific place on the basilar membrane. Fig. 1 is a reproduction of this illustration.

However, the hypothesis, originally initiated by Von Békésy [1] and again assumed by Lighthill, that the dimensions of the scala media – the endolymph filled duct that lies between, and actually separates the perilymph filled scala vestibuli and scala tympani from each other – can be ignored as a contributing factor in the hydrodynamic behavior of the perilymph movements inside the cochlea, is fundamentally wrong.
Within the perilymph content of the cochlea – no more than a tiny droplet – pressure differences cannot occur as a result of evoked sound signals, as Lighthill suggests in his theory. According to the laws of physics, we must remain aware of the fact that as soon as a local disturbance is evoked anywhere inside the perilymph fluid this disturbance will always propagate and expand with the speed of sound, which is 1500 m/s. Therefore, the existence of two sound energy transport phenomena with different transfer velocities within this tiny cochlear volume of perilymph fluid as suggested by Lighthill shown in Fig. 1, is impossible.

Studying Lighthill’s paper we may conclude that his mathematical exercise is a mere attempt at justifying the observations of other scientists during their experiments. He attempts to explain the observed mechanical activities inside the cochlea in terms of sound wave propagation by means of traveling waves. However, fundamental physical basis for this exercise is lacking.

The perilymph fluid in scala tympani and the scala vestibuli can in fact only move as a whole, in a push-pull movement caused by the excitation of the oval window. The shape of the narrow and folded duct that consists of both the scala tympani and the scala vestibuli, does not change this principle.

1.5 In conclusion

Taking all of the above-mentioned objections against the existing concepts into account we conclude that an alternative theory must exist. In this theory we do not only need to incorporate the existence and role of the scala media, but also the fact that the perilymph fluid in the scala vestibuli and the scala tympani moves as a whole, as it forms an incompressible fluid column.
2. The new hypothesis

2.1 Middle ear functioning

The varying pressure in the outer ear canal that is evoked by sound stimuli activates the eardrum. The eardrum, in turn, brings the ossicular chain into motion. This motion is transferred via the stapes – closely connected to the oval window – to the perilymph within the cochlea.

In our hypothesis we describe a different functioning of the middle ear. We propose that the musculus tensor tympani and musculus stapedius can modify the amplification ratio of these movements. A gradual increase of sound stimuli thus causes a gradual decrease within that ratio. Under normally varying sound conditions this adaptation is so subtle that it generally cannot be observed without the use of extremely sensitive experimental equipment.

This hypothesis seems realistic as, in the event of sudden loud sound bursts, the stapedius reflex – the only described function of the stapes until now – can actually be regarded as an extreme and instantaneous performance of this attenuation principle.

The continuous functioning of both of the middle ear muscles suggests that only in abnormal conditions, such as a sudden sound burst caused by a nearby explosion, might the protection mechanism for the delicate inner ear be switched on too late, and is thus rendered ineffective. In normal conditions however, this protection mechanism proves to be extremely effective.

The above-mentioned system functions as a damped mass-spring system activated by an externally generated sound signal as shown in Fig. 2.

The cooperative functioning of the eardrum and ossicular chain attenuation in Fig. 2 can be regarded as the amplifier $R_s$. The perilymph fluid column equals the mass $m$. The combined elastic behavior of the eardrum, oval window and round window are represented by the spring constant $k$, while the damping is indicated as $\xi$.

The perilymph velocity as a function of frequency can be calculated by means of a standard solution in physics: the second order differential equation. Depending on practical data for $m$, $\xi$, and $k$ in the cochlea, the results of this calculation will show a resonance frequency $f_r$ of 1000 – 2000 Hz and a 3 dB per octave increase in perilymph velocity $v$ for frequencies lower than $f_r$, and a 3 dB per octave decrease for frequencies higher than $f_r$. 
2.2 Relation between sound stimulus and electrical output in the cochlea

Entirely consistent with the findings of Wever and Lawrence [16], it is argued that direct sound stimuli on either the oval window or on the round window, i.e. the other membrane separating the middle ear from the cochlea, cause a similar change in the electrical cochlear potential related to the perilymph motion. The experiments that Wever and Lawrence describe in their paper titled “The acoustic pathways to the cochlea” – published in 1950 – are of much greater importance and hold more consequences for our understanding of the functioning of the hearing sense, than either one of them had originally anticipated or stated in the conclusions of their paper.

We believe that our comments with regard to the various statements in this publication of Wever and Lawrence are of crucial importance, and we have therefore decided it warrants quoting the abstract of this article, each part provided with our comments.

Quote:
--- Measurements are made of the relative effectiveness of the round window as a route of entrance of sounds to the cochlea. When the ear is normal this route is of no importance, but when the middle ear apparatus is absent and its advantage is lost to the oval window route a sound will have nearly equal access by both windows. ---
Comment:

In a normal ear the existence of the round window as a flexible partition between the scala tympani and the middle ear cavity is highly important as it enables the overall push-pull movements of the incompressible perilymph fluid in the combined scala vestibuli and scala tympani. Which is also part of the following findings of Wever and Lawrence.

Quote:

--- When the inner ear is reached by both pathways at once the cochlear potentials represent the vector sum of what would result from the two separate waves. As phase and intensity relations between the two pathways are altered the potentials pass through maximum and minimum values, which for equal intensities of the two waves vary from a 6 dB gain to a complete loss. Evidence is produced to show that each pathway of stimulation excites the same sensory cells and in the same intensity pattern. Over the major portion of the frequency range a minimum of response results when the waves are in phase as they enter the oval and round windows. ---

Comment:

The above-mentioned statements provide us with a substantial amount of data, allowing us to draw a few very important, however, previously unnoticed conclusions.

- Two sound pressure signals that are equal in intensity and that simultaneously attempt to push-pull the oval window and the round window, in the same direction in and out of the cochlea – causing window movements in the same phase – will result in a vector sum for the perilymph stimuli of zero. This means that the perilymph velocity is zero as well. Wever and Lawrence established that in this case the changes in cochlear potentials are zero. We may therefore conclude that the change in cochlear potentials is dependent on the resulting perilymph velocity and not on the pressure load.

- The vector sum of perilymph velocity stimulation reaches its maximum at the moment that oval and round windows are moving in opposite phase. And for stimuli equal in intensity on both windows this results in a doubled perilymph velocity. And because the 6 dB gain reported by Wever and Lawrence equals a factor of four, we may conclude that the associated change in cochlear potentials is squared.
The sound pressure stimulus is transferred to the perilymph velocity, which means that this part of the transfer function is the result of differentiation. The increased factor of four in cochlear potentials, related to the increased factor of two for the perilymph velocity, means that this part of the transfer function is acquired by squaring.

The cochlear potentials are proportional to the differentiated and squared sound pressure stimulus at the oval or round window.

The experiments of Wever and Lawrence are confirmed by the experiments of Voss et al. [17]. In their paper Voss et al. concluded:

--- These results are consistent with the hypothesis that the window pressure-difference is a dominant mechanism in producing a cochlear response; the contributions of any "non-series" connections between the windows e.g. the cochlear aqueduct or cochlear fluid compressibility, must be small. ---

Nowadays, these experimental results actually form the basic reference for active middle ear implantable devices, such as the Vibrant Soundbridge. Among others, recent studies with regard to the efficacy of these types of hearing aids presented by Beltrame et al. [18], show that they are very successful in the transfer of sound stimuli to the inner ear by means of stimulation of the round window via a so-called floating mass transducer.

However, Voss et al. [17] have apparently overlooked the even sharper conclusion that can be drawn: it is actually the velocity of the perilymph fluid within the cochlear duct that evokes the change in cochlear potentials. Naturally, this velocity is generated by the impact of a net force on the perilymph due to the difference in sound pressure affecting one or both windows.

In the practical case of the Vibrant Soundbridge the floating mass transducer actually evokes the force on the round window, which in turn sets the perilymph in the cochlea into motion.

2.3 Bone conduction

As the petrous bone is the toughest bone in the human body, it is practically impossible for deformation to occur through the vibrating movements that are caused by acoustic pressure changes. Therefore, there must be another means for this signal transfer. A bone conduction audiogram – apart from a reduced transfer of especially the higher frequencies – is not fundamentally different from a pure airborne tone-threshold audiogram. It follows that the perilymph movement within the cochlea
must somehow also generate the bone conducted signal transfer. And indeed there is such a possibility. The scala tympani is directly connected with the cerebrospinal cavity by means of the cochlear aqueduct, which has its opening in the vicinity of the round window. The cochlear aqueduct enables the perilymph to move backwards and forwards between the cranial cavity and the cochlear channels (scala tympani and scala vestibuli). The perilymph in this cranial cavity is exposed to the alternating pressures caused by the sound induced vibrations of the shell shaped cranial bones that are located in the front and the back of the skull.

As a consequence of Newton’s fundamental law of motion we know that when a larger amount of fluid inside the skull must be brought into motion, the effect of these stimuli will be smaller for the higher frequency contributions than for those of the airborne stimuli for similar frequencies.

Furthermore, the stapes introduces the airborne signal that results in a backwards and forwards motion of perilymph, in opposite phase to the back and forth motion of perilymph along the basilar membrane, which is evoked by bone conduction. This results in a slight reduction of the airborne signal, which actually is the stronger of the two stimuli.

The alternating movement of perilymph between the cerebrospinal cavity and the scala tympani via the cochlear aqueduct will also be partially directed towards the round window. This splitting up into two directions of the perilymph movement creates a reduced perilymph velocity in front of the basilar membrane, and finally results in a reduction of the bone conduction signal. The rate of reduction depends on the mobility ratio between the oval window and the round window. A further decrease in round window mobility, related to the mobility of the oval window, results in a higher perilymph movement in front of the basilar membrane, which evokes a higher bone conduction signal, while an increase would result in a lower one.

 Actually, this perilymph push-pull hypothesis is more than just a free claim. In their paper Stenfelt et al. [19] presented an overview of earlier investigations by other colleagues. From this overview, we list a number of previous investigations that clearly offer an indication for the perilymph push-pull hypothesis.

Freeman et al. [20] and Sohmer et al. [21] have shown that a sound pressure stimulus introduced in the cerebrospinal fluid is transmitted to the cochlea, where it evokes a hearing sensation even when there is insufficient vibration of the skull bones for it to be considered a normal bone conduction transmission.

Carlborg et al. [22], and Yoshida and Uemura [23], previously showed that a change in static pressure in the cerebrospinal fluid could be transmitted to the cochlear fluid. Even though, others have also disputed the influence of the cochlear aqueduct on the human sense of hearing. [24-28].
As early as 1936, Von Békésy [29] proposed that the aqueducts could transport fluid in and out of the cochlea. Tonndorf [30], when he experimentally immobilized the round window in cats found a reduction in the air conduction sensitivity of a mere 20 to 30 dB. He attributed the residual hearing to the incomplete immobilization of the round window. However, within the same study he reported that no significant change was found in either the bone conduction or the air conduction sensitivity when the round window was obstructed, as long as the cochlear aqueduct remained open.

Groen and Hoogland [31], in their examination of a patient diagnosed with round window otosclerosis, found that hearing by bone conduction deteriorates very little in the low frequency region, provided that the cochlear aqueduct and the inferior cochlear vein remain open. For increasing frequencies, they found thresholds to deteriorate by 6 dB/octave, which they attributed to the frequency dependent, increased impedance of the fluid canal.

Our hypothesis that bone conduction consists entirely of the push-pull movement of perilymph from the cerebrospinal cavity via the cochlear aqueduct, and is evoked by the vibration of the shell shaped bones of the skull, while the rigid temporal bone does not deform at all, is just one logical step further than the functional possibilities indicated by others.

2.4 The hydrodynamic behavior in the cochlea

2.4.1 Von Békésy’s traveling wave versus the Bernoulli effect

As we mentioned before, the statement by Von Békésy that the Reissner membrane between the scala vestibuli and scala media does not form an obstacle for pressure-induced flows of both perilymph and endolymph fluid, contains a fundamental misinterpretation. This statement is at variance with the laws of hydrodynamics, which do not allow for the substitution of the scala media, a separate three-dimensional tube filled with fluid, by a two-dimensional thin membrane.

The explanation of the different consequences in behavior for the calculation models that serve as analogue geometries for Von Békésy’s [1] two-channel model, and the actually existing three-channel model, can be seen in Fig. 3.

Although the perilymph fluid on either side of the scala media moves in opposite directions as shown in (A), in simplification (B), we see that the two pressure impacts $\Delta p$ on the basilar membrane, evoked by both fluid movements in scala vestibuli and scala tympani along the basilar membrane, will be identical in strength, but opposite in direction. This means they will cancel each other’s stimulus impact.
on the basilar membrane; the basilar membrane does not have a net stimulus and will remain at rest.

Only if the connection between scala vestibuli and scala tympani at the helicotrema would be significantly smaller than the average cross section of both scala vestibuli and scala tympani elsewhere in the cochlear duct – actually serving as a strong fluid throughput reduction valve – will stimulation of the oval window generate differences in pressure between the scala vestibuli and scala tympani, possibly causing movements of the entire scala media in turn.

Von Békésy really did generate movements of the entire scala media in his experiments with cochleae extracted from cadavers. He actually detected movements of the Reissner membrane through his observation of the silver particles that he had purposely scattered over the membrane, in order to make these movements visible. These movements, he concluded, were caused by high vibration stimuli.

However, the indirect observation of what appeared to be a traveling wave on the basilar membrane evoked by high vibration stimuli – actually movements of the silver particles scattered over the Reissner membrane – led Von Békésy to his hypothesis that the scala media and the basilar membrane directly follow the Reissner membrane.
Our model using three contributing compartments, as shown in (C), has a pressure stimulus $\Delta p$ on both the Reissner membrane and the basilar membrane, which forces both membranes to move in outward directions relative to the scala media because the endolymph fluid in this scala media is at rest, while the perilymph fluid in both the scala tympani and scala vestibuli moves with a velocity $v$, directed along the core of each of these two scalae.

According to Bernoulli’s law, this pressure difference on either side of both the Reissner membrane and basilar membrane is represented by:

$$\Delta p = -\frac{1}{2} \rho v^2$$  \hspace{1cm} (1)

here $\rho$ is the density in kg/m$^3$ and $v$ the velocity of the perilymph in m/s.

In media such as gasses and fluids a traveling sound wave is characterized by a series of areas, alternately higher and lower in pressure, traveling at the speed of sound. When we combine this with the fact that the perilymph, like most fluids is highly incompressible, it results in distances between those successive areas much larger than the length of the cochlear duct. The only possible conclusion is that the perilymph is push-pulled as a whole by the motion of the stapes and oval window and behaves like a moving fluid column. All scientific literature and observations in experiments to investigate this have shown the round window to be deflecting in opposite direction relative to the movement of the oval window.

Therefore, in our opinion the perilymph filled part of the cochlea, existing of scala tympani and scala vestibuli, functions as a flow conducting tube. According to the Bernoulli principle this flow creates decreases in pressure proportional to the square of the fluid velocity, multiplied by the fluid density. In essence, the squaring of the velocity causes the decreases in pressure along the basilar membrane to have both a static and a dynamic component. Finally, we can distinguish two functions in the combined action of the basilar membrane and the organ of Corti. First, that of a series of separate frequency selective pressure sensors, each with a high quality factor reacting to frequency signals that correspond with the local resonance frequency. This function is essential in hearing sounds. The combined action, in our opinion, functions as a pressure sensor for very low frequency pressure variations. This second sensor function can be crucial in the transfer control of the average signal via the eardrum and ossicular chain. Thus, the auditory system can be compared to a well-damped second order resonance system with a relatively low resonance frequency, which is adjustable in amplitude. Calculations of these effects will be provided in the following chapters.

Recently, a peer reviewer, upon consultation with a scientific journal editor, made a comment in his report that the Bernoulli effect is a non-linear higher order Navier-
Stokes phenomenon, which is extremely small and thus insufficiently strong to play a role in the process of sound transfer within the cochlea.

Quite to the contrary however, the Bernoulli effect in almost incompressible low viscous fluids that move inside a tube is rather a strong effect, thus the application of this effect inside the cochlea is correct indeed. The application of Bernoulli’s law in this case is based upon Newton’s law of conservation of energy – describing the balance between the amount of kinetic and potential energy. Denying the validity of this application of Bernoulli’s law is equal to ignoring one of the most fundamental laws of physics.

Further indisputable evidence exists for the application of Bernoulli’s law in case there is movement of the perilymph in the cochlear duct.

Data from literature shows that for normal sound stimuli the maximal displacements of the oval window are substantially smaller than 10 micrometers. However, for the sake of an extremely high hypothetical perilymph velocity calculation evoked by a 20 kHz sinusoidal vibration, let us start at the extreme amplitude $U_0$ of 10 micrometers. A simple calculation results in the maximum velocity $v_{pm} = 2\pi f U_0 = 1.26$ m/s.

In addition we can make use of the following data found in standard literature for the parameters that will play a role in this process:

a. For the perilymph duct in the cochlea, the channel diameter is given: $d = 0.3$ mm.

b. While the kinematic viscosity is indicated to be: $\nu = 0.801 \times 10^{-6}$ m$^2$/s.

c. Perilymph is regarded as a practically incompressible fluid, similar to water.

Using these data we calculate for the Reynold’s number $Re$:

$$Re = \frac{v_{pm} \times d}{\nu} = 471.9 \quad (2)$$

This result is only 1/5 of the critical value for a $Re$ between 2000 and 3000 that is required to change the flow conditions from laminar for low $Re$, to turbulent for high $Re$. Since $v_{pm}$ is proportional to frequency $f$, for lower frequencies $Re$ will be even smaller.

Hence the flow of the incompressible perilymph fluid thus remains laminar for all sound frequencies in the cochlea. It follows, that the parameters involved remain unchanged for all regular sound signals that generate alternating perilymph velocities.
Therefore, without any constraint the quasi-static approach can be used, which means replacing the constant velocity \( v_p \) with the time dependent velocity \( v_p(t) \).

For a single frequency \( f \) this results in the sinusoidal velocity relation:

\[
v_p(t) = (2\pi f U_0) \cos(2\pi ft)
\]

### 2.4.2 Transfer of sound signals in the cochlea and the Bernoulli effect

Suppose that in a tube such as the cochlear duct there is a flow with a uniform velocity \( v \). In each cross section perpendicular to the core of this duct, this flow generates a decrease in pressure \( \Delta p \) on the walls as shown in equation (1). First, it is noticed that there is no restriction for \( \Delta p \) with regard to the direction of the flow; secondly, we notice that all conditions for the validity of this equation are met within the cochlea. This means that if the stapes generates a time dependent displacement \( u(t) \) at the oval window over a cross sectional area \( S_0 \), this would correspond with a displacement in volume \( \Delta V(t) \) of the incompressible perilymph as shown in equation (4).

\[
\Delta V(t) = S_0 u(t)
\]

With the volume displacement of equation (4) we can calculate that at location \( x \) with local cross sectional area \( S_x \) the local velocity \( v_x(t) \) of the perilymph is described by:

\[
v_x(t) = \frac{S_0}{S_x} \frac{du(t)}{dt}
\]

where \( du(t)/dt \) is the time derivative of the displacement of the oval window, which equals the velocity of this displacement.

If we take a pure tone with sound pressure amplitude \( p_0 \) and frequency \( f \) for the sound stimulus, this results in a sinusoidal time dependent perilymph displacement as shown in equation (6):

\[
u(t) = Ap_0 \sin(2\pi ft)
\]

where \( A \) represents the transfer function from sound pressure in front of the eardrum to oval window displacement. Then, after a few calculations, namely differentiation for the transfer of displacement into velocity followed by squaring due to the Bernoulli effect, we will find, at location \( x \) with local cross sectional area \( S_x \) for the pressure decrease on the wall of the cochlear duct:

\[
\Delta p(t)_x = -2\rho(A\pi f p_0)^2 \left(\frac{S_0}{S_x}\right)^2 \cos^2(2\pi ft)
\]
This can be expressed in a combination of a constant stimulus and a double frequency stimulus contribution:

$$\Delta p(t)_x = -2\rho(A\pi f_0 p_0)^2 \left(\frac{S_0}{S_x}\right)^2 \cos^2(2\pi ft)$$

$$= -\rho(A\pi f_0 p_0)^2 \left(\frac{S_0}{S_x}\right)^2 [1 + \cos(4\pi ft)]$$  \hspace{1cm} (8)

From here, we shall make use of the so-called $1/f$ relation for sounds found in nature. By this $1/f$ relation, the sound pressure amplitude $p_{0i}$ of a pure tone in a tone complex will be reciprocal to its frequency $f_i$. Immediately in equation (8), the reason for the preference for $1/f$ sound contributions becomes clear: the signal strength, given by $f_i p_{0i}$, of each stimulus contribution on the basilar membrane becomes frequency independent.

This well-established $1/f$ quality of sounds is a phenomenon that is omnipresent in nature [32]. The mammalian auditory sense shows a perfect adaptation to such sounds.

Another result becomes clear when we observe that in similarly shaped cross sections, the evoked signal at location $x_1$, where for example the characteristic diameter of the cochlear duct is 2 times smaller than at location $x_2$, the cross section is actually 4 times smaller. This results locally in a perilymph velocity that is 4 times higher, whereas due to the squaring of this velocity, the stimulus on the basilar membrane is 16 times stronger in accordance with the Bernoulli effect. This offers an explanation for the fact that the tapered shape of the scala tympani, from the round window to the helicotrema, is beneficial to sensing lower frequencies.

Consequently, this explanation is different from the ‘whispering gallery’ phenomenon hypothesized by Manoussaki et al. [33]. Moreover, the ‘whispering gallery’ phenomenon is only valid if the wavelengths of the propagating waves are relatively small in relation to the dimensions of the observed curved gallery; a condition that does not exist within the cochlea at all. Furthermore, the cochlear model that Manoussaki et al. [33] applied as a basis for their mathematical calculations, using Laplace’s equation as a valid differential equation, consists of two, instead of three ducts.

A closer look at the result of equation (8) shows that a sound that consists of one frequency generates a time independent decrease in pressure on the walls of the cochlear duct, proportional to the square of the frequency, multiplied by the pressure amplitude in front of the eardrum. In addition, we see a time dependent harmonic change in pressure with an amplitude equal to the value of the constant pressure decrease.
However, this in time varying harmonic contribution has a doubled frequency. Expressed in musical terms, the original pure tone in a sound signal is transposed up by exactly one octave.

This forms the basis for an explanation of the double frequency response phenomena and other remarkable frequency shifts, like the ‘half octave shift’ phenomena that were first investigated and described by Davis et al. [34], followed by Mitchell et al. [35] and Cody and Johnstone [36,37], following their experiments with traumatized animals and humans.

When a complex sound wave composed of several Fourier components is offered via the ossicular chain, each of these components contributes accumulatively to the deflection of the oval window.

Therefore, equation (6) will be changed into a sum of sinusoidal contributions. As a consequence of the quadratic effect in the build-up of pressure on the wall, as shown in equation (9), and again using algebraic and trigonometric relations, it can be demonstrated that two frequencies $f_i$ and $f_j$ with respective perilymph velocity amplitudes $2A_i$ and $2A_j$, contribute to the constant pressure on the basilar membrane with $-\rho(A_i^2 + A_j^2)$. For the time dependent part of the pressure $\Delta p(t)$ the contributions are purely harmonic and, in case $f_j \geq f_i$, calculated as frequencies $2f_i, 2f_j, f_j - f_i, f_i + f_j$, with corresponding amplitude ratios respectively $\rho A_i^2, \rho A_j^2, 2\rho A_i A_j, 2\rho A_i A_j$. This is shown in equation (10).

$$\Delta p(t) = -2\rho A_i^2 \cos^2(2\pi f_i t) - 2\rho A_j^2 \cos^2(2\pi f_j t)$$

$$\Delta p(t) = -4\rho A_i A_j \cos(2\pi f_i t) \cos(2\pi f_j t)$$

$$\Delta p(t) = -\rho A_i^2 - \rho A_j^2 - \rho A_i^2 \cos(4\pi f_i t) - \rho A_j^2 \cos(4\pi f_j t) - 2\rho A_i A_j \cos\{2\pi(f_i - f_j)t\} - 2\rho A_i A_j \cos\{2\pi(f_i + f_j)t\}$$

As the principle, explained in equations (8) to (10), is valid for each combination of two Fourier frequency components in the spectrum of a sound wave, it can be calculated that for a sound wave that consists of 100 individual Fourier components, this will result in a maximum number of 10,000 Fourier components. In addition, there is a static contribution that consists of the total signal average over time. This average consists of the sum of the time independent terms in equation (10), as shown in equation (11).
Generally, the total number of frequencies in the sound energy spectrum is significantly reduced in sound waves with a substantial number of harmonic tones, because large numbers of the sum and difference frequencies are identical to, and will merge with, other harmonic contributions.

\[ \Delta p = -\rho \left( A_i^2 + A_j^2 \right) \] for 2 contributions

and

\[ \Delta p = -\sum_{i=1}^{N} \rho A_i^2 \] for \( N \) contributions

Upon closer examination, the four frequencies that result from the multiplying process of two original Fourier components \( f_i, f_j \), while \( f_j \geq f_i \), allow us to distinguish between a single or pitch frequency \( f_j - f_i \) and a triplet, formed by \( 2f_i, f_i + f_j, \) and \( 2f_j \), with frequency intervals equal to the corresponding pitch frequency. For each of the two original components, equal in perilymph velocity amplitude, we find that the resulting sum and difference frequencies of all the signal amplitudes on the basilar membrane are twice as high as the amplitudes of the corresponding doubled frequencies.

If \( f_j \) is chosen as the one octave higher frequency of \( f_i \), the octave twin-tone combination, this results in a remarkable series of frequencies \( f_i, 2f_i, 3f_i, \) and \( 4f_i \), with amplitude ratios \( 2 : 1 : 2 : 1 \). A perfect fit for a pitch with its successive overtones.

The above shows why this hearing theory matches the practical experience. These results are in accordance with Euler’s definition of ‘Gradus Suavitatis’: his mathematical explanation that describes the human experience of music; the various degrees of harmoniousness, consonance or pleasantness.

If the ear is stimulated with both harmonic and disharmonic tone complexes as shown in the residual pitch perception experiments by Schouten [38], De Boer [39] and others, it becomes clear that it is easy to evoke residual pitch in an unresolved tone residue.
3. Methods and experiments for verification

3.1 Stimulating the cochlea with composed frequency complexes

In order to verify the theoretically predicted auditory phenomena that result from the differentiating and squaring process in the scala tympani in front of the basilar membrane, we can use composed sound complexes for which the contributing frequency components possess two characteristic conditions.

The first of these conditions is that the average sound energy of an individual frequency contribution must be equal to that of all the other individual contributions. And as a consequence of equation (10), this can be achieved if we use a \( \frac{1}{f} \) dependency for the sound pressure amplitude.

The second condition is that apart from the imposed deviations of a few Hz that affect one or two components in the series, the successive frequencies must have equidistant positions on the frequency scale.

In keeping with these two conditions, and based on the equations (8) and (9), we can pre-calculate how all the energy frequency contributions in a sound complex are formed and combined out of the sound pressure signal that evokes the stimulus. We can also calculate how these energy frequency contributions subsequently stimulate the basilar membrane and the organ of Corti. And finally we can explore the calculated results in real sound experiments.

For this purpose Yves Mangelinckx, co-author of the Appendices has developed a relatively simple and easy-to-use, efficiently operating software program. For the calculations we have chosen to make use of a very high bit resolution in the sound signals; this in order to avoid any disturbing aliasing or unwanted beat phenomena that could otherwise occur in the compositions.

[See also Appendix A I or A II]

3.2 Pitch perception in incomplete harmonic sound complexes

For the complicated pitch perception example given by De Cheveigné [40] the corresponding and resulting sound energy frequency spectrum can be calculated by combining all the contributions with an identical frequency in the harmonic complex. This is shown in Fig. 4.

Again, because our auditory sense appreciates the \( \frac{1}{f} \) criterion for tone contributions in the sound pressure tone complex \( P_0 \), this is chosen as a constraint in Fig. 4a.
After differentiation, this results in a perilymph velocity frequency spectrum $v_0$ inside the scala tympani and scala vestibuli for all frequency contributions in equal perilymph velocity amplitudes, as shown in Fig. 4b. We also apply the harmonic contributions in such a manner that they all have zero phase differences.

Hence, all contributions in $P_0$ are purely sinusoidal and zero at time $t = 0$. The calculation of all primary and combination frequencies – sum and difference frequencies in this example – finally offers the resulting sound energy frequency spectrum $S_f$, given in Fig. 4c. We can also see in Fig. 4c that the smallest distance $\Delta f$ between successive harmonics, present eight times in the frequency spectrum, is equal to the fundamental $f_0$. The fundamental $f_0$ of the series is still absent in the sound energy spectrum of this example. However, a quick comparison by the observer of the pitch frequency with a pure tone reference, leads to $f_0$ as that reference frequency. This, due to the fact that the offered reference sound pressure stimulus will also undergo the differentiating and squaring process in the cochlea. This results in the doubled frequency $2f_0$ on the basilar membrane and will then sound similar to the fundamental frequency in the combination of $2f_0 + 4f_0 + 7f_0 + 9f_0$. 

Fig. 4. Transfer of sound pressure signal to sound energy frequency spectrum
When we execute the experiment as shown in Fig. 4, again using the $1/f$ constraint in a tone complex that consists of the 200+400+700+900 Hz frequencies, the additional offer of either a 99 Hz or a 101 Hz test frequency with a relative amplitude of 2 makes it easier to determine the pitch in the original tone complex.

In either case this results in a strong 2 Hz beat, as a consequence of the described process of differentiation and squaring in the aforementioned theory. This phenomenon has been, and indeed can be, heard by any listener with a relatively normal hearing ability. This is a clear indication that the doubled frequencies – 198 and 202 Hz, respectively – interfere with the 200 Hz frequency component of the tone complex on the basilar membrane, which evokes a stimulus of 199 and 201 Hz respectively, with a beat of 2 Hz.

For residual tone complexes – harmonic series where the first harmonic or fundamental is missing – the differentiating and squaring process in the cochlea perfectly generates the missing fundamental. The findings of Monahan et al. [41], show that the listener reconstructs the fundamental or virtual pitch within roughly 100 ms after the sound signal onset, which is in accordance with earlier electrophysiological research. According to their paper titled “Neuromagnetic evidence for early auditory restoration of fundamental pitch” it also suggests that this fundamental pitch is reconstructed at the very beginning of the auditory cortex.

Our experiments, however, clearly show that this reconstruction process takes place even earlier. In our experiments we found that the pre-calculated signals in the basilar membrane are perceived exactly as predicted. Therefore, contrary to the conclusion by Monahan et al. that an early neural mechanism is responsible for the mystery of the inferential pitch, we have strong evidence that the cause for this reconstruction of the virtual or fundamental pitch is hydrodynamic in origin.

### 3.3 Residual pitch perception in enharmonic tone series

The experiment with enharmonic tone series described by De Boer [39] is carried out again, this time with the additional constraint that all frequency components in the series have relative amplitudes that correspond with the $1/f$ criterion. An example of such an enharmonic tone complex consists of the following frequencies: 1400+1600.5+1800 Hz.

In the transformation from this sound pressure signal in front of the eardrum to the sound energy signal on the basilar membrane, the $1/f$ amplitude ratio criterion that is imposed on the sound pressure frequency contribution, leads to the following set of frequency contributions with accompanying relative amplitudes in the sound energy frequency spectrum:
From the triplet, the in frequency doubled primaries: 2800+3201+3600 Hz, with amplitudes at relative value 1. The three sum frequencies: 3000.5+3200+3400.5 Hz, with amplitudes at relative value 2. And the three difference frequencies with amplitudes at relative value 2, as shown in equation (12):

\[ 2\cos[2\pi(200.5t)]; 2\cos[2\pi(199.5t)]; 2\cos[2\pi(400t)] \]  

(12)

The first two contributions in equation (12) differ by 1 Hz, which is smaller than the frequency resolving properties of the cochlea and thus cannot be heard separately. With their identical amplitude they can be combined according to the following equation.

\[ 2\cos[2\pi(200.5t)] + 2\cos[2\pi(199.5t)] = 4\cos[2\pi(200t)]\cos[2\pi(0.5t)] \]  

(13)

As a result a listener will hear the 200 Hz fundamental frequency of the harmonic residual tone complex, which consists of three frequencies 1400+1600+1800 Hz, with an amplitude modulation of \[4\cos[2\pi(0.5t)]\]. Moreover, during the 2-second period of this 0.5 Hz modulation, the 200 Hz signal reaches twice its maximum value 4. This implies that the fundamental frequency is heard with a strong 1 Hz beat, exactly as De Boer [39] reported in his thesis.

### 3.4 Addition of harmonics and their influence on beat phenomena

A theoretical consequence of this new pitch perception model is that the expansion of the triplet with another two harmonics under the 1/f criterion forms the series: 1200+1400+1600.5(or 1599.5 Hz)+1800+2000 Hz, which results in a different beat phenomenon.

Calculation shows that the final result of equation (12) will change into:

\[ 4\{1 + \cos[2\pi(0.5t)]\}\cos[2\pi(200t)] \]  

(14)

Now, during the period of 2s the modulation factor \(4\{1 + \cos[2\pi(0.5t)]\}\) varies only once between 0 and its maximum 8. This means that the beat in the 200 Hz fundamental is still strong, but with a 0.5 Hz frequency, halved in comparison to the original experiment by De Boer [39].

Experiments with these residue sound complexes that are composed according to this principle can be experienced by any listener, and positively prove this theoretical result in beat phenomena. We would like to point out that this phenomenon is an unknown anomaly in the current hearing theory, as it cannot be explained nor predicted from the existing pitch theories.
Moreover, by changing the harmonic combinations it can experimentally be proven that the decrease in beat frequency from 1 Hz to 0.5 Hz affects the frequency of 200 Hz, commonly named the missing fundamental. Thus the change in beat frequency from 1 Hz into 0.5 Hz will also be heard when we add the harmonic contributions of 1000+1200 Hz respectively 2000+2200 Hz to the triplet of 1400+1600.5+1800 Hz, again using the $1/f$ criterion as a constraint.

Analytical calculations of the energy frequency spectra of the other two combinations show that only the 1 Hz beat signal within the 200 Hz missing fundamental of the triplet (as shown in equation 13) changes into a 0.5 Hz beat signal. (see equation 14)

### 3.5 Modifying a beat frequency by adding a low frequency stimulus

A third and very convincing experiment that offers strong evidence for a differentiating and squaring cochlea is based on the addition of only one single lower frequency $f_{\text{l}}$ to the 1400+1600.5+1800 Hz triplet,. With a properly calculated frequency and amplitude, prescribed by this new theoretical concept, the additionally generated contribution to the missing fundamental of 200 Hz will change the existing 1 Hz beat in the triplet into a 0.5 Hz beat in the four tone experiment.

Here again, the $1/f$ criterion must be met, in addition to an extra multiplier factor of 2 for the lower frequency $f_{\text{l}}$. This, in order to obtain the correctly weighted amplitude contribution to the combined fundamental of the 200 Hz signal on the basilar membrane. Finally, the lower frequency of 100 Hz $f_{\text{l}}$ must be chosen as 100 Hz, as the aforementioned squaring process for this singular frequency contribution will result in a doubled frequency contribution on the basilar membrane.

This will result in the addition of the correct 200 Hz stimulus to the basilar membrane; necessary for the change in the 1 Hz beat due to the modulation factor $4 \cos[2\pi(0.5t)]$ according to equation (13) to the beat of 0.5 Hz due to the modulation factor $4(1 + \cos[2\pi(0.5t)])$ according to equation (14). A singular 200 Hz sound pressure contribution however, would generate a 400 Hz stimulus on the basilar membrane, and therefore is not capable of changing the beat frequency that exists in the 200 Hz fundamental.

The execution of this simple experiment shows that when the amplitude is calculated according to this new theoretical concept, the pre-calculated 100 Hz contribution changes the beat of 1 Hz heard in the 1400+1600.5+1800 Hz triplet into 0.5 Hz heard in the 100+1400+1600.5+1800 Hz sound fragment, while the 200 Hz signal in a 200+1400+1600.5+1800 Hz sound fragment does not change the beat at all. Precisely as predicted!
3.6 Schouten’s explanation for the strike note of bells must be revisited again

A remarkable result of these experiments is that they also provide a logical explanation and solution for the mystery of the strike note of bells. This mysterious phenomenon of auditory perception – already observed by Rayleigh [42], investigated by Jones [43] and later thought to be explained by Schouten et al. [44]: that when one strikes a bell the rapidly dampened frequency that is initially heard, is actually perceived to be one octave lower than the expected pitch or fundamental frequency of that bell. When we apply the following principles we can finally explain this phenomenon. In all previous investigations [42-44] the strike note frequency of bells is compared with a pure reference frequency $f_r$ generated by a tuning fork or a tuned frequency generator.

However, due to the aforementioned squaring process in the cochlea, the reference frequency $f_r$ will always evoke a sound energy contribution with a doubled stimulus frequency $2f_r$ on the basilar membrane, a fact that is not recognized in the common hearing theories thus far. As a consequence, the frequency of the strike note of a bell – actually the most prominent existing frequency among the partials in the sound energy spectrum – that stimulates the listener’s basilar membrane, is identical to the doubled frequency $2f_r$ rather than the reference frequency $f_r$ of the sound pressure stimulus.

3.7 Is pitch shift in an enharmonic tone complex with equidistant frequencies an illusion?

As early as 1929, Fletcher [45] mentioned an experiment that describes a phenomenon which never occurs in nature, and that only can be conducted artificially in the laboratory. He reported that the musical quality of the original tone in a harmonic tone complex that is enhanced with an extra 30 Hz – which creates a completely inharmonic tone complex – will, consequentially, be destroyed. In 1940, Schouten [46] described a similar experiment in which the fundamental and a few of the other lower numbered harmonics were missing. He found that the residue heard displayed an upward pitch shift in the direction of the corresponding frequency shift. De Boer [39] reproduced this experiment and has discussed the results extensively.

A consulted auditory expert would have us believe that this perceived pitch shift – clearly subjective observations that were reported by trained examiners – could not be explained by our concept of a differentiating and squaring auditory sense.

According to the experiments described by De Boer the pitch can, by good approximation, be perceived to be equal to the sub harmonic frequency of the center fre-
frequency of the complex, which is closest to the difference frequency. For example: in
the
\[ 1430+1630+1830+2030+2230+2430+2630 \text{ Hz} \]
tone complex, for a center frequency of 2030 Hz and a difference frequency of 200 Hz, one tenth of the 2030 Hz of the center frequency, equals 203 Hz.

Therefore, in order to attain less subjective experimental data with regard to that which actually transpires, we conducted a series of similar experiments by means of more advanced equipment and with the addition of the following extra features. For the amplitude distribution in the tone complex we used the $1/f$ constraint, and we inserted a ‘pitch tracing mechanism’ in the form of a twin-tone that generated either a pitch identical to the expected pitch, or one that was a few Hz higher or lower.

In the first experiment we have used the tone complex:

\[ 1400+1600+1800+2000+2200+2400+2600 \text{ Hz} \]

This obviously is a harmonic tone complex with 200 Hz as fundamental or first harmonic. In this tone complex the first six harmonics, including the fundamental, are missing. Based on our concept, calculations according to the principle explained in equation (10) result in a value of 12 for the relative amplitude of the evoked fundamental of 200 Hz. For such a harmonic tone complex it is well known that the fundamental is equal to the pitch.

Therefore, if we want to create an identical pitch of 200 Hz that has the same amplitude as the other tone complex on the basilar membrane, and we make use of the corresponding first and second harmonics, the 200 and the 400 Hz, under the $1/f$ constraint, we must apply a relative amplitude to both of $\sqrt{6} = 2.449$.

However, if instead, we introduce either the twin-tone complex 201+402 Hz or the 199+398 Hz, these twin-tones evoke a 201 Hz pitch, respectively 199 Hz pitch. In combination with the 200 Hz pitch of the 7-tone harmonic complex, the combinations of both pitches will result in a 200.5 Hz pitch, respectively 199.5 Hz pitch, both with a beat of 1 Hz. This will be heard exactly as calculated. Even for very low levels of the sound signal intensity.

For the next experiment we used the enharmonic tone complex:

\[ 1430+1630+1830+2030+2230+2430+2630 \text{ Hz} \]

We are not trained in music, so as examiners we were not able to distinguish the pitch and its shift upward, as reported by others. In our experience the sound is rather harsh.
However, as De Boer [39] explained in his thesis this pitch can best be compared to that of the harmonic tone complex:

\[ 1421 + 1624 + 1827 + 2030 + 2233 + 2436 + 2639 \text{ Hz}. \]

Here according to former results the pitch is equal to 203 Hz, the first harmonic of this series. Therefore, we have tried to search for this pitch by means of the pitch tracing mechanism. We first tried to approach the twin-tone 203+406 Hz. However, instead of adding an extra intensity to the pitch we clearly heard a 3 Hz beat. When we used the twin-tone 202+404 Hz we heard a beat of 2 Hz, and in case of the twin-tone 201+402 Hz we heard a 1 Hz beat. The beat disappeared completely for the twin-tone of 200+400 Hz and finally, in the case of the twin-tone 199+398 Hz the beat returned to 1 Hz.

As no examiner can find any trace of a pitch shift in these beat experiments, the only possible conclusion is that the pitch shift that is heard by trained listeners in music perception is an illusion. The actual pitch remains at 200 Hz, which is the smallest difference frequency in the enharmonic series.

3.8 Infrasound can be heard according to the squaring principle

In an experiment with an enharmonic triplet with frequencies 1400+1600.5+1800 Hz, we extended this triplet with an extra pair of harmonics either at the low end of the triplet, at both ends or at the high end, in order to form a 5-tone complex.

In this 5-tone complex the 1 Hz beat in the audible pitch of 200 Hz, which is evoked on the basilar membrane by the triplet, changes into a 0.5 Hz beat in the same pitch of 200 Hz. This manipulation of a beat is not restricted to pitch frequencies above the 20 to 30 Hz threshold of audibility; it is also possible to manipulate a beat for pitch frequencies much lower than this threshold.

If we use two frequencies, for example 600+610 Hz, and impose the \(1/f\) amplitude constraint we will hear a tone of 605 Hz with a beat in the form of a vibrato of 10 Hz. This phenomenon is commonly and erroneously explained as the beat effect caused by the combined nearby frequencies. The following series of experiments clearly establishes that this is not the case.

If we apply a triplet that consists of a 600+610+620 Hz frequency in a sound fragment we will still hear this as a 10 Hz vibrato, but now in a tone of 610 Hz. However, if we change the 610 Hz frequency in this triplet into either 609 or 611 Hz the vibrato of 10 Hz is heard with an extra 2 Hz beat. This is the result of a modulation of the 10 Hz difference frequency between zero and the maximum amplitude within an interval of 0.5 seconds.
We can make this clear if we add one of the following combinations: 580+590; 590+630; 630+640 Hz to the triplet of 600+609(or 611)+620 Hz. In that case, we hear a similar effect as previously described in section 3.5. The 2 Hz beat in the vibrato of 10 Hz changes into a 1 Hz beat.

Still, we have to consider that the only frequency that can be modulated here is the 10 Hz pitch.

The same effect occurs when we add another triplet, for instance 700+710+720 Hz to the triplet of 600+609(or 611)+620 Hz. These two triplets together create four difference frequency contributions with equal amplitudes, namely: 9 Hz, 11 Hz and two contributions of 10 Hz. These four contributions together combine to a 10 Hz frequency signal with a beat of 1 Hz, and are therefore halved compared to the beat in the first experiment.

That this is what actually happens can be established as we change the 710 Hz in the added triplet into 709 or 711 Hz. In that case the 1 Hz beat in the 10 Hz vibrato again changes into a 2 Hz beat. However, if we change the phase of the 709 or 711 Hz to 180° the 10 Hz beat disappears almost completely and an unmodulated 20 Hz vibrato is heard instead.

This is because the two difference frequencies of 9 Hz and 11 Hz that are evoked out of the 600+609(or 611)+620 Hz triplet, and the same two difference frequencies of 9 Hz and 11 Hz that stem from the 700+709(or 711)+720 Hz triplet, result in pairs with practically equal amplitudes, that are opposite in phase. They almost completely cancel each other’s contributions. What remains is the combination of two contributions of the difference frequency of 20 Hz from the combinations 620 – 600 and 720 – 700 Hz, which is heard as a 20 Hz vibrato.

The experimental results of all these examples will undoubtedly have made you aware of three important fundamental facts, namely that:

- only the pitch frequency can be modulated by means of a distinct mistuning of one or more of the higher contributions.
- this pitch frequency corresponds with the difference frequency between the successive higher contributions in the series.
- there is no indication of a pitch shift when the frequency of all the contributions within the series is shifted equally in the number of Hz.
4. The role of DC signals in the organ of Corti and the cochlear amplifier

If we use realistic values for the various quantities we can calculate the extent of the pressure effect on the basilar membrane. Deflections in the eardrum measure approximately 0.1 micrometer. For a deflection with a frequency of 1000 Hz, while the density of the perilymph is estimated to be the same as that of water, 1000 kg/m³, and 1:1 is given for the amplitude amplification in the ossicular chain, the constant pressure will be 0.1 mPa. This results in a factor of 5 above 0 dB SPL, which is $2\times10^{-5}$ Pa. If the amplification factor equals the estimated ‘pressure transduction’ in the ossicular chain of approximately 25, the pressure load on the basilar membrane increases by a factor 625, due to squaring. These values are easily detected, certainly when sufficient resonance is present in the basilar membrane to evoke the required signals in the auditory nerve.

This mechanism further implies that a constant sound pressure signal also causes a constant pressure difference on both sides of the basilar membrane, as shown in equation (10). The subsequent pressure decrease in the scala tympani results in a pulling force on the entire surface of the membrane; this causes the basilar membrane to move away from the tectorial membrane.

An increase by a factor of two in the level of any constant sound pressure signal in the outer ear channel will result in a corresponding increase of the constant pressure level on the basilar membrane by a factor of four. This can be seen in equation (10) if all $A_i$ and $A_j$ values are replaced with $2A_i$ and $2A_j$ values, respectively.

With this model a candidate for the function of logarithmic ‘loudness control’ in our auditory system is found. In the static pressure signal, all the contributions of the original Fourier components are present and reflected as the sum of the quadratics of all amplitudes, as shown in equation (11). Upon closer examination it becomes clear that this equation represents the combined average sound energy exactly.

The average sound signal constantly changes in time, because each individual perilymph velocity amplitude contribution $2A_i(t)$ ($i = 1$ to $N$) can independently or in combination with others change in time. This is expressed by the sum of the $N$ time dependent terms in equation (15).

$$\Delta p(t) = -\frac{1}{4} \sum_{i=1}^{N} \rho A_i^2(t)$$ (15)
The existence of this ‘quasi static’ pressure signal on the basilar membrane leads to our hypothesis that it may function as a feedback signal, controlling loudness, by altering the tonus of the muscles in the middle ear. Generally, this relatively slow changing signal can be extracted from all the other; typically AC alternating signals that are heard through a low pass filter. The eardrum and ossicular chain acquire a setting that realizes a higher threshold for hearing sensitivity, proportional to the average sound induced pressure on the basilar membrane.

It is known that the tightening of a membrane can significantly reduce its deflection. For the eardrum this means, that by contraction of the musculus tensor tympani the range reduction of signal transfer may vary with an approximate factor 30. Moreover, the setting of the musculus stapedius can also reduce the transfer factor of the lever in the ossicular chain, between the incus and the stapes, by another factor of approximately 30 times. When each of these muscles individually generates maximum signal transfer reduction factors – as a consequence of the average level of the sound energy signal – the combined ratio in deflection between the eardrum and the oval window, which is a product of both factors, will be reduced by a factor of roughly 1000. Finally, this results in a velocity decrease of 1000 times in the perilymph compared to the velocity in the tympanic membrane. However, the pressure that is exerted on the basilar membrane, as a consequence of squaring due to the Bernoulli effect, will be reduced by a ratio of deflection of one million, which equals 60 dB. Completely analogue to the adapting iris diaphragm – which adjusts to increased or decreased light intensity reaching the eye – this automatic sound energy feedback control system continuously protects our auditory sense at the best possible location: at the entrance of the sensory system.

Here, the effect of a 60 dB dynamic range can be observed. In a quiet environment the hearing threshold is slightly above 0 dB. In the utmost tolerable sound environment, the threshold is 60 dB. Within the current theory this factor of 60 dB is what is expected from the ‘cochlear amplifier’. In our model, however, the amplifier is situated in the middle ear and not in the cochlea, even though the change in the DC component in the cochlear microphonics serves as the detection signal.

One question that remains to be answered by means of careful experiments is the fact that both the inner and the outer hair cells react to the average sound intensity signal. The inner hair cells however, do not react as strongly to the frequency signals that are caused by local resonance, as the outer hair cells do. As a consequence, the outer hair cells are more vulnerable to frequency dependent damage than the inner hair cells. If the inner hair cells are responsible for the signal in the mechanism that controls the attenuation of the eardrum and the ossicular chain, in response to the varying incoming sound energy, then the entire auditory mechanism will be less vulnerable to possible overload by frequency dependent signals.
5. The organ of Corti as a highly selective frequency analyser

The common hypothesis is that only the inner hair cells are expected to generate signals that reach the auditory nerve. This hypothesis is based on the idea that the basilar membrane moves towards the tectorial membrane in reaction to the occurring waves of pressure, higher than the ambient pressure that normally exists in the cochlear duct (scala tympani and scala vestibuli). The long-held assumption is that this movement of the basilar membrane causes the development of shear forces that stimulate the inner hair cells. Whereas, the inner hair cells are very sensitive to these shear forces, according to Hudspeth et al. [47]. In our functional concept however, the pressure that is evoked in the cochlear duct (scala tympani and scala vestibuli) is not a wave, but an overall pressure that is lower than the ambient pressure. This merely causes the basilar membrane to bend away from the tectorial membrane. Therefore, the presumed development of shear forces that could stimulate the inner hair cells is clearly out of the question.

The tectorial membrane lies completely enclosed in the scala media and apart from its connection to the stiff bony center axis of the cochlea; it is surrounded by endolymph fluid at rest. When its morphology and ultra structure are taken into consideration, the tectorial membrane is more likely to function as a relatively non-deformable position reference. Consequently, the tectorial membrane cannot be a moving object. The arrays of outer hair cells are embedded in the basilar membrane at those places along the core spiral of the cochlear duct, where due to pressure stimuli the largest local displacements of the basilar membrane are to be expected.

Moreover, the top of a hair bundle that is part of the outer hair cell is anchored in the tectorial membrane. Therefore, these hair bundles undergo stress forces when the basilar membrane moves away from the tectorial membrane, as a result of the under-pressure – caused by the Bernoulli effect – that is evoked in the scala tympani through the back and forth movement of the perilymph.

Because the electrical current inside the hair cells is responsible for evoking the signal contribution in the auditory nerve, the direct interconnection of every ten afferent axons of nerve cells that are connected to the outer hair cell is such that a parallel switching of these electrical current sources is attained. Careful experiments may prove that indeed, the outer hair cells cooperate in order to evoke a much stronger combined signal to the auditory cortex. Moreover, this parallel switching leaves the organ of Corti less vulnerable to individual hair cell collapse or local damage.
As stated, the Reissner membrane and the basilar membrane bend away from the tectorial membrane and therefore, instead of shear forces that affect the hair bundles of the inner hair cells, stress forces are generated that affect the outer hair cells. The inner hair cells, however, are situated along a strip close to the edge where the basilar membrane is attached to the solid wall. This means that the inner hair cells are located in places where they can hardly be affected by vibrations or deflections of the basilar membrane. Thus, apart from the relatively small displacements that are mainly induced by unwanted mechanical distortions, the inner hair cells probably only evoke signals to compensate for the influence of these distortions of the signal. Therefore, the number of inner hair cells can be much smaller than that of the outer hair cells, reduced by a factor 10, actually.

Moreover, we would like to introduce the following contribution; an article by Leibbrandt [48] that was first published in 1966, which only very recently came to our attention. During our contact with the author he informed us that this article received very little acknowledgement or response from the scientific community at that time. In our opinion, a significant injustice to this paper and the findings presented therein. We would, therefore, like to share the entire abstract from this publication with you.

--- In guinea pigs cochlear microphonic responses were studied during stimulation with harmonic high tone complexes. In the apical portion of the cochlea a sine wave with the frequency of the "missing fundamental" could be recorded. The amplitude of this low frequency microphonic potential (CM) in the third turn of the cochlea appears to be about equal to the amplitude of the strongest component of the high tone complex recorded in the basal turn. The "missing fundamental" appears to stimulate the apical portion of the cochlea, which indicates cochlear analysis according to a certain place principle, although apparently not the Fourier principle.---

In this publication Leibbrandt reports that aside from the presence of the missing fundamental, he also found the performance of the sum frequencies of the sound pressure stimuli.

The experimental results that were reported in this publication clearly confirm our assertion. The electronic signals that are evoked in the organ of Corti closely resemble the in frequency components split sound energy signal.
6. Resonance phenomena in the basilar membrane

Similar to the primary signal transfer in the middle ear, the basilar membrane also locally behaves as a damped spring-mass system. This means that the Fourier components, with frequencies close to the local resonance frequency in the pressure load on the basilar membrane, will produce larger movements of the membrane and consequently evoke a higher frequency signal. This signal can be strongly influenced by the damping factor in the local situation. It is a commonly held misconception that a system in liquid does not – or can hardly – come into resonance. The perilymph liquid present in the scala tympani and the endolymph liquid in the scala media, on either side of the basilar membrane, entirely surround the basilar membrane and the organ of Corti with fluid, thus creating the so-called ‘underwater piano’. However, two tone-masking experiments [49] indicate the existence of place related tuning with an average amplification factor at resonance of 10^6, which equals 60 dB. The reason for this 60 dB resonance is found in these experiments. The masking frequency signal, which does not lead to resonance within the test frequency resonance zone, must be at least 50 – 60 dB higher than that of the test signal that does lead to resonance.

The construction of the auditory system – this precise combination of the geometry and the use of materials, including the fluids – provides conditions that allow the system to freely resonate and vibrate. After that, it is only a matter of correct tuning.

It is obvious that quite a number of Fourier frequency components in the pressure stimulus, especially those in the higher range, have such high frequencies that the basilar membrane no longer has any appropriate regions with the required resonance frequencies available, to transfer their doubled frequency signals to the auditory nerve.

These frequencies are simply out of range for the ‘resonator window’, which can be defined as the total range of frequencies that the basilar membrane is sensitive to. Yet, their existence can be detected as long as they are capable of sufficiently moving the perilymph fluid inside the cochlear duct. When combined with high frequencies that cannot be heard, these frequencies remain capable of creating difference frequencies that are audible again. In the squaring process these frequencies can even, in combination with other audible higher frequencies, create both barely audible sum frequencies and better audible difference frequencies. They continue to contribute to a triplet that is considered to be incomplete.

For example, if in equation (10) the highest frequency \(2f_j\) is too high to be a stimulus for the basilar membrane, the other two frequencies in the triplet \(2f_i\) and \(f_i + f_j\) can still, together with the difference frequency \(f_j - f_i\), be present in the total signal.
However, the contribution with frequency $2f_j$ in equation (10) does no longer contribute to the signal in the organ of Corti. In principle, both the frequency and the amplitude of the missing component can be reconstructed using equation (10).

If the brain receives all this information, a simple experiment will show that the existence of the highest frequency can be detected. For instance, switching this signal component off causes only the $f_i$ to remain and both of the combination frequencies to disappear. The principle described above also explains why an unresolved harmonic high tone complex, with a pitch $\Delta f$ that is too low in relation to the used frequencies, still contributes to the total loudness of the fundamental. Not all successive frequencies in the tone complex can be distinguished separately; however, their common pitch is still evoked as a part of the sound energy frequency spectrum. Still, these beat phenomena can even be heard when we compose high frequency sound fragments.

For example, again using the $1/f$ constraint as a condition, it can be shown that even though none of the frequencies in the composed triplet of 7800+8000+8200 Hz can be heard as a separate tone, the change from the 8000 Hz frequency in the triplet to 8001 (or 7999) Hz will cause the listener to hear a 2 Hz beat in a rather unpleasant, shrill and piercing sound, dominated in timbre by the higher frequencies. Adding another triplet with the $1/f$ constraint, such as 500+700+900 Hz, not only changes the shrill and piercing sound into a more acceptable sound but it also changes the 2 Hz beat into a 1 Hz beat, while the final resetting of the 8001(or 7999) Hz to 8000 Hz results in the complete disappearance of the beat phenomenon.

Moreover, in this new model, on the low end of the audible frequency spectrum, two very close frequencies, for instance 100 and 103 Hz with equal amplitudes, (generally explained as a resulting 101.5 Hz tone with a beat of 3 Hz), will create an audible triplet on the basilar membrane with frequencies of 200, 203, and 206 Hz, as well as a non-audible difference frequency of 3 Hz.

In this audible triplet the amplitude of the central frequency of 203 Hz is twice as high as the amplitude of each of the two other contributions of 200 and 206 Hz, respectively. Generally, based solely on this combination of frequencies, it is impossible to distinguish these three different tones in the triplet separately.

However, when using goniometric formulas the two tones of 200 and 206 Hz can be combined. This results in another frequency contribution of 203 Hz with amplitude equal to the already existing 203 Hz component, but with a 100% amplitude modulation of 3 Hz, similar to that which is shown in equation (13).

Both contributions to the 203 Hz tone together finally deliver a signal of 203 Hz with a strong 3 Hz beat, similar as is shown in equation (14).
7. Basilar membrane resonance phenomena instead of traveling waves

The other reason for the extremely high frequency selectivity is based on the high quality factor in the resonance performance of the basilar membrane. The relative peak deflection value for a particular resonance frequency can be calculated using the general equation for amplitude of a second order resonance system. In the case of a 1000 Hz resonance frequency \( f_c \) and an amplification factor of 60 dB this would result in 0.0003 Hz. Much smaller than the 3 Hz that is actually found.

Therefore, apart from the influence of the detection of sum and difference frequencies, caused by switching one of the frequency contributions on and off, the quality factor in the resonance performance also offers a clearly recognizable triplet of frequencies. The frequencies in this triplet are separated from each other, as the peak width of each of the three closely adjacent frequencies is significantly smaller than their mutual distance on the frequency scale.

If we calculate the phase relations of that same second order resonance system with the equation of phase, we find that for membrane resonance frequencies higher than the stimulus frequency, the phase of the membrane movements equals the phase of the stimulus frequency. For membrane resonance frequencies that are lower than that of the stimulus frequency, the movements of the basilar membrane show a retarded phase shift of 180°. The phase for the basilar membrane movement at center frequency is retarded over 90°. This means that the auditory nerve receives the final signal, almost exclusively, from the contributions in the center frequency region. The contributions of the two flanks however, cancel each other due to their identical amplitude and opposite phase.

This mathematical calculation shows for the logarithmically distributed local resonance frequencies \( f_c \) of the basilar membrane, the response characteristic that Ren [50] observed in his experiments on gerbils: a very restricted symmetrical local movement phenomenon, which travels along the basilar membrane.

In our opinion this phenomenon is erroneously interpreted as evidence for a ‘traveling wave’ along the basilar membrane. We argue that it is not a traveling wave, but a ‘phase wave’, that consists of coherent place dependent phase shifted local reactions to a stimulus that is simultaneously present throughout the basilar membrane.

Fig. 5. shows successive basilar membrane movements in a series of sequential steps, \( T/12 \) of one complete convolution of a harmonic vibration, which is the typical deflection profile in the region of the center frequency \( f_c \).
Fig. 5. Deflection profiles of the basilar membrane around $f_c$ in sequential steps of $T/12$

In early experiments Von Békésy [1] applied energetically high vibration stimuli, which inevitably evoked harmonic disturbances. Such disturbances consequently generate higher harmonic phase waves elsewhere on the basilar membrane. These simultaneously generated higher harmonic phase waves are located within distinct areas – determined by their frequency sensitivity – between the round window and the area on the basilar membrane that is active as a result of the stimulus by the
fundamental $f_c$. Since these higher harmonics are coherent in both phase and amplitude with their fundamental frequency $f_c$, the corresponding phase waves on the basilar membrane will also be coherent. Consequently, if an applied stroboscope flashes in the same rhythm as the fundamental frequency $f_c$, these harmonics will be frozen in an identical shape as well. This easily leads to the erroneous observation that there is more than just one ‘convolution’ of a wave, which strengthens the idea of an enlarged ‘traveling wave’.

Moreover, in a combined paper Wever, Lawrence and Von Békésy suggested that they were aware of the fact that a traveling wave, that carries sound energy and transfers this energy to the basilar membrane, may not exist [2] inside the cochlea.
8. Conclusions

Our careful analysis of the results that were obtained in the auditory pathway experiments that were executed by Wever and Lawrence [16], leads us to the conclusion that the auditory sense differentiates and squares the incoming sound pressure signal. As a result the basilar membrane and the organ of Corti are stimulated with a signal that is proportional to the sound energy signal.

This can only be explained by the application of the Bernoulli effect. And thus, we can not only clarify the human appreciation of $1/f$ sound stimuli; we can also explain the existence of both DC and AC components in the cochlear potential, as well as the logical preference for harmonic tone complexes.

The differentiation and squaring of incoming sound signals also offers a mathematically determined mechanism that firmly places the pitch perception of the high contributions of combination frequencies within the cochlea, even in the case of residual and unresolved harmonic tone complexes.

It has been established that the rigid cochlear envelope is hardly deformable. This leads to the realistic hypothesis that the ‘bone conducted’ sound signals evoke stimuli by means of a similar process to those of the airborne sound stimuli, namely by means of a push-pull movement of the perilymph, in this case, out of the cerebrospinal cavity via the cochlear aqueduct.

Furthermore, the envelope of the cochlear potential that is proportional to the average sound energy signal – when filtered by a low pass filter and in conjunction with the attenuation properties of the eardrum, ossicular chain and middle ear muscles – provides a high dynamic gain control system. As such, this control system does not only serve as a protection device for the delicate structures of the inner ear, it also serves as an automatic loudness control mechanism with a 60 dB gain factor.

Indeed, our new model provides a very realistic alternative for the currently held belief that the cochlear amplifier would be located within the inner ear, actually, a hypothesis for which very little evidence has ever been found.

As evidence, we offer you the mathematically predictable examples of composed sound fragments that evoke stimuli in the cochlea, stimuli that – even though not detectable in the sound pressure signal itself – can still be heard and verified by any listener. This strongly confirms our proposed theory, which in our view convincingly explains the functioning of the auditory sense.

Another expert of pitch phenomena, in a private communication with us, stated that our concept could not explain all the basic residual pitch phenomena. These thorough residual pitch experiments however, clearly show that quite the opposite is
true. We are convinced that our exercise in residual pitch phenomena is very successful indeed. Especially so, as anyone – even those that are not trained in music or sound perception – should be able to complete these experiments successfully.

Therefore, we are convinced that our cochlear model provides a clear solution to the still existing problems that were also mentioned in Alain de Cheveigné’s historical overview entitled: "Pitch perception models from origins to today" which is published on the Internet [51].

As De Cheveigné’s conclusions in this paper of 2004 are so closely correlated to our findings, we cannot possibly withhold the following citation:

--- Modern ideas reincarnate older ideas, and their roots extend as far back as records are available. Models that are in competition today may have common roots. The historical approach allows commonalities and differences to be put in perspective. Hopefully this should help to defuse sterile controversy that is sometimes harmful to the progress of ideas. It also may be of use to newcomers to the field to understand, say, why psycho-acousticians insist on studying musical pitch with unresolved stimuli (that sound rather unmusical), why they add low pass noise (which makes tasks even more difficult), etc. The good reasons for these customs are easier to understand with a vision of the debates from which present-day pitch theory evolved. ---

Our revision and study of the entire set of mechanisms and functions – actually a new and exciting paradigm – enables us to explain most, if not all of the thus far unsolved, mysteries in the functioning of the auditory sense.
9. Acknowledgements

We, the authors Pim Heerens and Sander de Ru, are very grateful to Gaby de Neef for her careful translation of the manuscript. We would like to thank her for the helpful suggestions she offered, and not in the least for her patience and understanding during the many hours of work.

We also would like to mention the valuable work of Mirjam Vonk. After reading and re-reading she commented on the readability of this booklet for educated laymen. She corrected a great number of redaction, lay-out, punctuation and typing errors. Besides, as a member of the ‘Auditory Brainstorm Group’, she truly was a magician in finding Pim relevant literature in the available sources on the Internet.

We are also deeply indebted to Yves Mangelinckx, another member of the ‘Auditory Brainstorm Group’. He cleverly developed the software program that enabled the composition and examination of the sound fragments for the experiments with the pre-calculated residual pitch and beat phenomena. Thanks to Yves, our readers can now verify these experiments after downloading his program from the Internet.

We would also like to mention the substantial support that Wijnandt de Vries offered during our first attempts at promoting our insights and ideas.

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Sander, would like to take this opportunity to express his gratitude to his father Jaap de Ru for his continued support, and his often wise and helpful comments during the many hours he spent reading the many versions of this manuscript.

A special word of thanks goes out to Sander’s colleague otolaryngologist Paul Struyvenberg for lending an ear whenever necessary during these past years, and to Colonel J. de Graaf, Commander of the Central Military Hospital, for allowing Sander the freedom to carry on with this scientific project.
Epilogue

You have just worked your way through our model that describes a new way of looking at the intricate workings of the human auditory sense. A model – actually a new hearing theory – that offers solutions for the unsolved anomalies, mysteries and questions, which have continued to baffle and bother many otolaryngologists and other experts in the field of audiology during these past 150 years.

A new theoretical concept emerges. It radically alters the way we presume that our sense of hearing functions. This publication provides thereto-important principles that form the basis for a new understanding of the auditory sense.

Unfortunately, we were not able to achieve publication in the customary scientific journals. We are convinced however, that our work can be of great importance to further research into the nature and function of the auditory sense, as well as its dysfunction. Ultimately, leading to the reconsideration of existing therapies and the development of new methods to treat hearing impairment and other disorders of the auditory sense.

In essence, we felt compelled to publish our work privately and we will be highly content if it were to launch much-needed scientific discussion. We sincerely hope that this new concept will stimulate many researchers specialized in the fields of audiology and otology to combine and compare any experimental results, that they have available from their own research, with our hearing theory. We also hope that our theory will inspire specialized researchers to develop new experiments that can further verify our statements. We are convinced that in doing so auditory researchers will realize the importance of this new theory for their respective fields of expertise.

It has taken us many years to arrive at this first independent publication. You may rest assured though; this is by no means the end. We have only just begun. In a second publication we will further delve into the auditory phenomena and anomalies that could thus far not be explained within the current models. Fortunately, we have found that within our theory, we are able to consistently provide seamless and logical explanations for these mysterious phenomena.

A rough outline for those chapters is taking shape. Polishing these diamonds in the rough however, will still take a fair amount of time. Heerens and I have therefore chosen to first publicize the foundation of our model; we hope that this publication will find its way to many researchers and colleagues. That it may initiate an open and honest debate, which will hopefully convince many of you that, indeed, it is high time for a paradigm shift.

Jacob Alexander de Ru

November 2010
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Appendices

Appendix I

Verification of calculations of residual pitch and beat phenomena

Willem Chr. Heerens, Yves Mangelinckx & J. Alexander de Ru

This program is ONLY available for computer systems running under Windows XP or Windows Vista

A I.1 Introduction

Appendix I and the associated software – a calculation program designed by Yves Mangelinckx to be downloaded from the Internet – together provide you with a tool to personally verify the predicted residual pitch and beat phenomena described in Chapter 3 of this booklet.

You are invited to download the software via one of the following websites:

http://www.een-andere-kijk-op-horen.nl/

or

http://www.a3ccm-apmas-eakoh.be/

This software program forms the experimental basis of the above-mentioned chapter and enables you to verify the beat and residue experiments that were explained in Chapter 3.

For each experiment described in these paragraphs: simply fill in the corresponding frequencies within the program, start the calculation. You will subsequently be able to listen to the sound fragment. The sound fragments can be heard via standard audio software programs like Windows Media Player or iTunes.

This program can easily be installed on systems running under Windows XP or Vista.

A I.2 How to use the software program after the installation

After downloading and installing this program – in a separate directory – proceed with the following 8 steps:
1. Open the directory of the program and click on the executable file named *Perception_Calculations.application*, the program fill-in screen pops up. [See for the screen display also Fig. A.1]

2. Select the duration of the sound complex to be heard in the first line. Steps of 5 seconds from 05 / 10 [default] / 15 / 20 / 25 / 30 seconds. [However we advise you to use 20 seconds in experiments where you have to count the number of beats in that period]

3. Fill in the frequency components in the boxes indicated as \( f_0 \); \( f_1 \); \( f_2 \); etc… [Boxes not in use remain empty. Maximum number of frequencies 10]

4. Select the amplitude criterion for each frequency by switching \( \uparrow \). [1/\( f \) amplitude [default] for equal energy contribution or \( \text{amplitude 1} \) for equal sound pressure contribution]

5. Fill in per frequency the extra multiplication factor to be used for the amplitude in the left box column. [Extra multiply Amplitude 1 [default] or >1]

6. Select sine or cosine function for each frequency by switching \( \uparrow \). [sin [default] or \( \cos \)]

7. Select the desired phase for each frequency by switching \( \uparrow \). [0 degree [default], otherwise between 0° and 360°]

8. Select how you want to hear the composed sound-complex or to store the wavelet on your PC with the buttons as shown below:

   For standard built-in sound machine under Windows XP click on:
   
   ![Build wave and Play. !XP Only! (Play within this form).](image)

   For sound machine installed on your PC click on:
   
   ![Build wave and Play. (Play: System.Diagnostics.Process).](image)

   For storing a wavelet on your PC click on:
   
   ![Write wave as ...](image)

   and follow further instructions.
**Remark**

Which character to use in this program for the decimal mark depends on your computer system. If your system has installed a dot decimal mark notation apply a dot. If your system has installed a comma decimal mark notation apply a comma instead of a dot.

![Lay-out of pop-up fill-in screen](image)

**A I.3 Possible calculations this program allows you to make**

Of course, you can vary all the frequencies in the given examples to convince yourself that the choice of frequency combinations actually isn’t that important as long as the differences between successive frequencies are equal. Because the pitch that you hear is built up from the combined difference frequencies.
You must also be sure to insert the appropriate change of the original value when a frequency disturbance of a few Hz from that equidistance is needed for the beat experiment. Inserting a series of 5 frequencies $f_0, f_1, f_2, f_3, f_4$, in the computation display, while all other settings remain at default, will lead to the calculation of the sound fragment $S$ built up by a Fourier series as function of time:

$$S = \frac{A}{f_0} \sin(2\pi f_0 t) + \frac{A}{f_1} \sin(2\pi f_1 t) + \frac{A}{f_2} \sin(2\pi f_2 t) + \frac{A}{f_3} \sin(2\pi f_3 t) + \frac{A}{f_4} \sin(2\pi f_4 t)$$

A I.4 The successive experiments

For your convenience, the following experiments have all been numbered and named according to their previous descriptions (experiments) in Chapter 3.

3.2. Pitch perception in incomplete harmonic sound complexes

Experiment 3.2.0

When you fill in the frequencies:

- $f_0 = 200 \text{ Hz}$; $f_1 = 400 \text{ Hz}$; $f_2 = 700 \text{ Hz}$ and $f_3 = 900 \text{ Hz}$

you hear a harmonic tone.

Experiment 3.2.1

To the frequencies in Experiment 3.2.0 you can add the extra tone:

- $f_4 = 99 \text{ Hz}$ or $f_4 = 101 \text{ Hz}$ with Extra Multiply Amplitude 2

[which is the test signal for pitch determination]

and you will hear a beat of 2 Hz.

[Counting 20 dips during a 10 second sound fragment means $20:10 = 2 \text{ Hz beat}$]

3.3. Residual pitch perception in enharmonic tone series

Experiment 3.3.0

When you fill in the frequencies:

- $f_0 = 1400 \text{ Hz}$; $f_1 = 1600 \text{ Hz}$; $f_2 = 1800 \text{ Hz}$

you will hear a harmonic tone.
**Experiment 3.3.1**

When you modify the center frequency:

\[ f_1 = 1600 \text{ Hz into } f_1 = 1600.5 \text{ Hz} \]

the triplet becomes:

\[ f_0 = 1400 \text{ Hz; } f_1 = 1600.5 \text{ Hz; } f_2 = 1800 \text{ Hz} \]

you will hear the same harmonic tone as in Experiment 3.3.0, but with a 1 Hz beat.

[Counting 10 dips during a 10 second sound fragment means 10:10 = 1 Hz beat]

**Experiment 3.3.2**

When you modify the center frequency:

\[ f_1 = 1600 \text{ Hz into } f_1 = 1599.5 \text{ Hz} \]

the triplet becomes:

\[ f_0 = 1400 \text{ Hz; } f_1 = 1599.5 \text{ Hz; } f_2 = 1800 \text{ Hz} \]

you will hear the same harmonic tone as in Experiment 3.3.0, again with a 1 Hz beat.

[Counting 10 dips during a 10 second sound fragment means 10:10 = 1 Hz beat]

**3.4. Addition of harmonics and their influence on beat phenomena**

**Experiment 3.4.0**

To the triplet in Experiment 3.3.2 you can add another contribution of two frequencies:

\[ f_3 = 1200 \text{ Hz and } f_4 = 2000 \text{ Hz} \]

to create the 5-tone complex:

\[ f_0 = 1400 \text{ Hz; } f_1 = 1599.5 \text{ Hz; } f_2 = 1800 \text{ Hz; } f_3 = 1200 \text{ Hz; } f_4 = 2000 \text{ Hz} \]

then you will hear a harmonic tone with a 0.5 Hz beat.

[Counting 5 dips during a 10 second sound fragment means 5:10 = 0.5 Hz beat]

**Experiment 3.4.1**

To the triplet in Experiment 3.3.2 you can add two other frequency contributions:

\[ f_3 = 1000 \text{ Hz and } f_4 = 1200 \text{ Hz} \] instead of: \( f_3 = 1200 \text{ Hz and } f_4 = 2000 \text{ Hz} \)

and again you will hear a harmonic tone with a 0.5 Hz beat, however with another timbre.
**Experiment 3.4.2**

To the triplet in Experiment 3.3.2 you can add two other frequency contributions:

\[ f_3 = 2000 \text{ Hz} \quad \text{and} \quad f_4 = 2200 \text{ Hz} \]

instead of:  

\[ f_3 = 1000 \text{ Hz} \quad \text{and} \quad f_4 = 1200 \text{ Hz} \]

and again you will hear a harmonic tone with a 0.5 Hz beat, however with yet another timbre.

**Remark**

*In case you find it difficult to distinguish and count a 0.5 Hz beat frequency you can also use:*

\[ f_1 = 1601 \text{ Hz} \quad \text{or} \quad f_1 = 1599 \text{ Hz} \]

*all beat phenomena will then be doubled in frequency.*

*The beat in the triplets will be heard as 2 Hz, while this 2 Hz beat will change into 1 Hz in the 5-tone complexes.*

---

**3.5. Modifying a beat frequency by adding a low frequency stimulus**

**Experiment 3.5.1**

When you fill in the triplet:

\[ f_0 = 1400 \text{ Hz}; \quad f_1 = 1601 \text{ Hz}; \quad f_2 = 1800 \text{ Hz} \]

you will hear a harmonic tone complex with a beat of 2 Hz.

**Experiment 3.5.2**

To the triplet in Experiment 3.5.1 you can add the test signal:

\[ f_3 = 100 \text{ Hz} \quad \text{with Extra Multiply Amplitude} \quad \text{2} \]

[which is the test signal for pitch determination]

you will hear the former harmonic tone complex – with a beat of 2 Hz – change into a harmonic tone complex with a much lower timbre and with a 1 Hz beat.

**Experiment 3.5.3**

To the triplet in Experiment 3.5.1 you can add the test signal:

\[ f_3 = 200 \text{ Hz} \quad \text{with Extra Multiply Amplitude} \geq \sqrt{2} = 1.4142 \]

and you will hear the former harmonic tone complex change in timbre, however the beat of 2 Hz doesn’t change.
3.7. Is pitch shift in an enharmonic tone complex with equidistant frequencies an illusion?

**Experiment 3.7.0**

When you fill in the frequencies of the 7-tone complex:
\[ f_0 = 1400 \text{ Hz}; f_1 = 1600 \text{ Hz}; f_2 = 1800 \text{ Hz}; f_3 = 2000 \text{ Hz}; f_4 = 2200 \text{ Hz}; \]
\[ f_5 = 2400 \text{ Hz} \text{ and } f_6 = 2600 \text{ Hz} \]
you hear a harmonic tone.

**Experiment 3.7.1**

To the 7-tone complex in Experiment 3.7.0 you can add the twin-tone contribution:
\[ f_7 = 201 \text{ Hz}; f_8 = 402 \text{ Hz}; \]
with an Extra Multiply Amplitude of \( \sqrt{6} = 2.449 \)
to create the 9-tone complex:
\[ f_0 = 1400 \text{ Hz}; f_1 = 1600 \text{ Hz}; f_2 = 1800 \text{ Hz}; f_3 = 2000 \text{ Hz}; f_4 = 2200 \text{ Hz}; \]
\[ f_5 = 2400 \text{ Hz}; f_6 = 2600 \text{ Hz}; f_7 = 201 \text{ Hz} \text{ and } f_8 = 402 \text{ Hz} \]
and then you will hear a harmonic tone with a lower timbre than in Experiment 3.7.0
and a 1 Hz beat.

**Experiment 3.7.2**

To the 7-tone complex in Experiment 3.7.0 you can add the twin-tone contribution:
\[ f_7 = 199 \text{ Hz}; f_8 = 398 \text{ Hz}; \]
with an Extra Multiply Amplitude of \( \sqrt{6} = 2.449 \)
and then you will hear an almost identical harmonic tone as in Experiment 3.7.1,
again with a 1 Hz beat.

**Experiment 3.7.3**

When you fill in the frequencies:
\[ f_0 = 1430 \text{ Hz}; f_1 = 1630 \text{ Hz}; f_2 = 1830 \text{ Hz}; f_3 = 2030 \text{ Hz}; f_4 = 2230 \text{ Hz}; \]
\[ f_5 = 2430 \text{ Hz} \text{ and } f_6 = 2630 \text{ Hz} \]
you will hear an enharmonic tone.

Remark

*Listeners who are trained in music are able to distinguish the pitch and its shift upward, as reported by De Boer and others.*

*In our experience the sound is rather harsh.*

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Experiment 3.7.4
In his thesis [A I 1] De Boer has explained that the pitch of Experiment 3.7.3 can best be compared to the pitch of the harmonic tone complex for which you can fill in:

\[ f_0 = 1421 \text{ Hz}; f_1 = 1624 \text{ Hz}; f_2 = 1827 \text{ Hz}; f_3 = 2030 \text{ Hz}; f_4 = 2233 \text{ Hz}; \]
\[ f_5 = 2436 \text{ Hz} \text{ and } f_6 = 2639 \text{ Hz} \]

and then you will hear a harmonic tone.

Remark

*Here – according to previous results – the pitch is equal to 203 Hz, the first harmonic of this series.*

Experiment 3.7.5
To the 7-tone complex in Experiment 3.7.4 you can add the twin-tone contribution:

\[ f_7 = 203 \text{ Hz}; f_8 = 406 \text{ Hz}; \text{ with an Extra Multiply Amplitude of } \sqrt{2} = 2.449 \]

to create the 9-tone complex:

\[ f_0 = 1421 \text{ Hz}; f_1 = 1624 \text{ Hz}; f_2 = 1827 \text{ Hz}; f_3 = 2030 \text{ Hz}; f_4 = 2233 \text{ Hz}; \]
\[ f_5 = 2436 \text{ Hz}; f_6 = 2639 \text{ Hz}; f_7 = 203 \text{ Hz} \text{ and } f_8 = 406 \text{ Hz} \]

and you will hear a harmonic tone with a lower timbre than in Experiment 3.7.4, without a beat.

Remark

*Therefore, we will attempt to search for this pitch in the enharmonic 7-tone complex of Experiment 3.7.3, by means of the pitch tracing mechanism. This mechanism consists of the addition of several different twin-tones to this tone complex, and to observe the changes in evoked beats.*

*In Experiment 3.7.6 we will start with the twin-tone of 203 Hz and 406 Hz because we can expect the pitch of 203 Hz De Boer has predicted.*

*And if he is correct you will not hear a beat with this twin-tone test.*
Experiment 3.7.6
To the 7-tone complex in Experiment 3.7.3 you can add the twin-tone contribution:
\[ f_7 = 203 \text{ Hz}; f_8 = 406 \text{ Hz}; \] with an Extra Multiply Amplitude of \( \sqrt{6} = 2.449 \) to create the 9-tone complex:
\[ f_0 = 1430 \text{ Hz}; f_1 = 1630 \text{ Hz}; f_2 = 1830 \text{ Hz}; f_3 = 2030 \text{ Hz}; f_4 = 2230 \text{ Hz}; \\
\] \[ f_5 = 2430 \text{ Hz}; f_6 = 2630 \text{ Hz}; f_7 = 203 \text{ Hz} \text{ and } f_8 = 406 \text{ Hz} \]
and then you will hear an enharmonic tone with a lower timbre than in Experiment 3.7.3. However, instead of hearing an extra intensity to the pitch as is expected, you will clearly hear a 3 Hz beat.

Experiment 3.7.7
When you replace the added twin-tone in Experiment 3.7.6 by the twin-tone:
\[ f_7 = 202 \text{ Hz}; f_8 = 404 \text{ Hz} \]
you will hear the same timbre as in Experiment 3.7.6, but with a beat of 2 Hz.

Experiment 3.7.8
When you replace the formerly added twin-tone by: \( f_7 = 201 \text{ Hz}; f_8 = 402 \text{ Hz} \)
you will hear a beat of 1 Hz.

Experiment 3.7.9
When you replace the formerly added twin-tone by: \( f_7 = 200 \text{ Hz}; f_8 = 400 \text{ Hz} \)
the beat disappears completely.

Experiment 3.7.10
When you replace the formerly added twin-tone by: \( f_7 = 199 \text{ Hz}; f_8 = 398 \text{ Hz} \)
the beat you will hear returns to 1 Hz.

Remark

As no examiner can find any trace of a shift in pitch in these beat experiments, the only possible conclusion is that the pitch shift, which is observed and reported by trained listeners in music perception, actually is an illusion. The actual pitch remains unchanged at 200 Hz, which is the smallest difference frequency in the enharmonic tone series.
3.8. Infrasound can also be heard according to the squaring principle

Experiment 3.8.0
When you fill in the twin-tone frequencies:
\[ f_0 = 600 \text{ Hz} \quad \text{and} \quad f_1 = 610 \text{ Hz} \]
you presume to hear a tone of 605 Hz with a vibrato of 10 Hz.

Remark
This is commonly explained as the beat effect of the combination of the two near-by frequencies. However that is not how it works, as can be seen in the following series of experiments.

Experiment 3.8.1
When you fill in the triplet frequencies:
\[ f_0 = 600 \text{ Hz}; \quad f_1 = 610 \text{ Hz} \quad \text{and} \quad f_2 = 620 \text{ Hz} \]
you will hear a tone of 610 Hz with a beat in the form of a vibrato of 10 Hz.

Experiments 3.8.2 – 3.8.3
When you change the \( f_1 = 610 \text{ Hz} \) in either 609 Hz or 611 Hz you create the triplet:
\[ f_0 = 600 \text{ Hz}; \quad f_1 = 609 \text{ Hz} \quad (\text{or} \quad f_1 = 611 \text{ Hz}) \quad \text{and} \quad f_2 = 620 \text{ Hz} \]
you will hear a tone of 610 Hz with a 10 Hz vibrato, and an extra beat of 2 Hz.

Experiments 3.8.4 – 3.8.5 – 3.8.6
When you add one of the following combinations to the triplet in Experiment 3.8.2 or 3.8.3:
\[ f_3 = 580 \text{ Hz} \quad \text{and} \quad f_4 = 590 \text{ Hz} \]
or: \( f_3 = 590 \text{ Hz} \quad \text{and} \quad f_4 = 630 \text{ Hz} \)
or: \( f_3 = 630 \text{ Hz} \quad \text{and} \quad f_4 = 640 \text{ Hz} \)
you will hear that the extra 2 Hz beat is changed into a 1 Hz beat.

Remark
You also have to consider that the only frequency that can be modulated is the 10 Hz pitch.
Experiment 3.8.7
To the triplet in Experiment 3.8.2 or 3.8.3 you can add another triplet:

\[ f_3 = 700 \text{ Hz}; \quad f_4 = 710 \text{ Hz} \quad \text{and} \quad f_5 = 720 \text{ Hz} \]

then you will hear that, apart from a change in timbre, the extra 2 Hz beat is changed into a 1 Hz beat.

Remark

*The combination of the four difference frequency contributions of 9; 2 times 10 and 11 Hz results in a signal of 10 Hz with a beat of 1 Hz. Halved compared to the beat in Experiment 3.8.2 or 3.8.3.*

Experiment 3.8.8
In the added triplet in Experiment 3.8.7 you can change the frequency \( f_4 = 710 \text{ Hz} \) into \( f'_4 = 711 \text{ Hz} \) and this triplet becomes:

\[ f'_3 = 700 \text{ Hz}; \quad f'_4 = 711 \text{ Hz} \quad \text{and} \quad f_5 = 720 \text{ Hz} \]

and you will hear that the extra 1 Hz beat again changes into a 2 Hz beat.

Experiment 3.8.9
When you change the phase of the \( f_4 \) into 180° in Experiment 3.8.8 you will hear that the 10 Hz beat disappears almost completely.

Instead of that you hear a not modulated 20 Hz vibrato.

Remark

*This occurs because the two difference frequencies of 9 Hz and 11 Hz (evoked out of the 600 + 609 (or 611) + 620 Hz triplet) and the same two difference frequencies of 9 Hz and 11 Hz (evoked out of the 700 + 709 (or 711) + 720 Hz triplet) are – pair by pair – practically equal in amplitude and have an opposite phase. Therefore they cancel each other’s contributions almost completely.*

*What remains is the combination of two contributions of the difference frequency of 20 Hz as a result from the combinations 620 – 600 and 720 – 700 Hz, heard as a 20 Hz vibrato.*

References

Appendix II

The residual pitch and beat phenomena to be heard in practice

Willem Chr. Heerens, Yves Mangelinckx & J. Alexander de Ru

Direct presentation of composed sound fragments
as a result of our experiments

A II.1 Introduction

In case you are not able to use the calculation program mentioned in Appendix I, this Appendix II and the associated sound fragments, calculated by us with the program designed by Yves Mangelinckx, provide you with the possibility to listen to the predicted residual pitch and beat phenomena as described in Chapter 3 of this booklet.

For each experiment described in Chapter 3 we have filled in the correct frequencies within the calculation program, and composed a sound complex fragment with a ten second duration.

You are invited to download the sound fragments via one of the following websites:

http://www.een-andere-kijk-op-horen.nl/
or

http://www.a3ccm-apmas-eakoh.be/

The fragments are grouped per subject in zipped directories.

In the report of these experiments below as an example the notation: [Sound E 2 0] refers to the file ( E 2 0.wav) in the list of sound fragments while E 2 corresponds to paragraph 3.2.
A II.2 Experiments

3.2. Pitch perception in incomplete harmonic sound complexes

Experiment 3.2.0
In this Experiment we have filled in the frequencies:

\[ f_0 = 200 \text{ Hz}; \ f_1 = 400 \text{ Hz}; \ f_2 = 700 \text{ Hz} \text{ and } f_3 = 900 \text{ Hz} \]
you will hear a harmonic tone in the sound fragment:

[Sound E 2 0]

Experiment 3.2.1
To the frequencies in Experiment 3.2.0 we have added:

\[ f_4 = 99 \text{ Hz} \text{ or } f_4 = 101 \text{ Hz} \text{, with Extra Multiply Amplitude 2} \]
[which is the test signal for pitch determination]
you will hear a beat of 2 Hz in the sound fragments:

[Sound E 2 1] respectively [Sound E 2 2]
[Counting 20 dips during a 10 second sound fragment means 20:10=2 Hz beat]

3.3. Residual pitch perception in enharmonic tone series

Experiment 3.3.0
In the calculation program we have filled in the frequencies:

\[ f_0 = 1400 \text{ Hz}; \ f_1 = 1600 \text{ Hz}; \ f_2 = 1800 \text{ Hz} \]
with that combination you will hear a harmonic tone in the sound fragment:

[Sound E 3 0]

Experiment 3.3.1
In Experiment 3.3.0 we have modified the center frequency from:

\[ f_1 = 1600 \text{ Hz into } f_1 = 1600.5 \text{ Hz} \]
the triplet becomes:

\[ f_0 = 1400 \text{ Hz}; \ f_1 = 1600.5 \text{ Hz}; \ f_2 = 1800 \text{ Hz} \]
and you will hear the same harmonic tone, but with a 1 Hz beat in the sound fragment:

[Sound E 3 1]
[Counting 10 dips during a 10 second sound fragment means 10:10=1 Hz beat]
Experiment 3.3.2

In Experiment 3.3.0 we have modified the center frequency from:

\[ f_1 = 1600 \text{ Hz} \quad \text{into} \quad f_1 = 1599.5 \text{ Hz} \]

the triplet becomes:

\[ f_0 = 1400 \text{ Hz}; \quad f_1 = 1599.5 \text{ Hz}; \quad f_2 = 1800 \text{ Hz} \]

and you will hear the same harmonic tone as in Experiment 3.3.0, but again with a 1 Hz beat in the sound fragment:

[Sound E 3 2]

[Counting 10 dips during a 10 second sound fragment means 10:10=1 Hz beat]

3.4. Addition of harmonics and their influence on beat phenomena

Experiment 3.4.0

To the triplet in Experiment 3.3.2 we have added another contribution of two frequencies:

\[ f_3 = 1200 \text{ Hz}; \quad f_4 = 2000 \text{ Hz} \]

which creates the 5-tone complex:

\[ f_0 = 1400 \text{ Hz}; \quad f_1 = 1599.5 \text{ Hz}; \quad f_2 = 1800 \text{ Hz}; \quad f_3 = 1200 \text{ Hz}; \quad f_4 = 2000 \text{ Hz} \]

now you will hear a harmonic tone with a 0.5 Hz beat in the sound fragment:

[Sound E 4 0]

[Counting 5 dips during a 10 second sound fragment means 5:10=0.5 Hz beat]

Experiment 3.4.1

To the triplet in Experiment 3.3.2 we have added two frequency contributions:

\[ f_3 = 1000 \text{ Hz}; \quad f_4 = 1200 \text{ Hz} \quad \text{instead of:} \quad f_3 = 1200 \text{ Hz}; \quad f_4 = 2000 \text{ Hz} \]

again you will hear a harmonic tone with a 0.5 Hz beat, however with another timbre in the 5-tone sound fragment:

[Sound E 4 1]

Experiment 3.4.2

To the triplet in Experiment 3.3.2 we have added two other frequency contributions:

\[ f_3 = 2000 \text{ Hz}; \quad f_4 = 2200 \text{ Hz} \quad \text{instead of:} \quad f_3 = 1000 \text{ Hz}; \quad f_4 = 1200 \text{ Hz} \]

again you will hear a harmonic tone with a 0.5 Hz beat, again with another timbre in the 5-tone sound fragment:

[Sound E 4 2]
Remark

_In case you find it difficult to distinguish and count a 0.5 Hz beat frequency we can also use, for instance:_

\[ f_1 = 1601 \text{ Hz} \quad \text{or} \quad f_1 = 1599 \text{ Hz} \]

all beat phenomena will then be doubled in frequency.

_For instance the beat in the triplet:_

\[ f_0 = 1400 \text{ Hz}; f_1 = 1601 \text{ Hz}; f_2 = 1800 \text{ Hz} \]

will be heard as 2 Hz as in the sound fragment:

[Sound E 4 3]

_Whilst by adding two extra frequencies:_

\[ f_3 = 1200 \text{ Hz}; f_4 = 2000 \text{ Hz} \]

we can create the 5-tone complex:

\[ f_0 = 1400 \text{ Hz}; f_1 = 1601 \text{ Hz}; f_2 = 1800 \text{ Hz}; f_3 = 1200 \text{ Hz}; f_4 = 2000 \text{ Hz} \]

and the 2 Hz beat you will hear is changed into 1 Hz in the sound fragment:

[Sound E 4 4]

### 3.5. Modifying a beat frequency by adding a low frequency stimulus

**Experiment 3.5.1**

We have filled in the triplet:

\[ f_0 = 1400 \text{ Hz}; f_1 = 1600.5 \text{ Hz}; f_2 = 1800 \text{ Hz} \]

so you will be able to hear a harmonic tone complex with a beat of 1 Hz in the sound fragment:

[Sound E 5 1]

**Experiment 3.5.2**

To the triplet in Experiment 3.5.1 we have added the test signal:

\[ f_3 = 100 \text{ Hz with Extra Multiply Amplitude 2} \]

[which is the test signal for pitch determination]

you can hear that this tone complex – which had a beat of 1 Hz – is changed into a tone complex with much lower timbre and with a 0.5 Hz beat in sound fragment:

[Sound E 5 2]
**Experiment 3.5.3**

To the triplet in Experiment 3.5.1 we have added the test signal:

\[ f_3 = 200 \text{ Hz with Extra Multiply Amplitude } \geq \sqrt{2} = 1.4142 \]

you will hear that the former harmonic tone complex is changed in timbre, however the beat of 1 Hz you hear, isn’t changed in the sound fragment:

[Sound E 5 3]

**3.7. Is pitch shift in an enharmonic tone complex with equidistant frequencies an illusion?**

**Experiment 3.7.0**

In the first one of this series we have filled in the frequencies of a 7-tone complex:

\[ f_0 = 1400 \text{ Hz; } f_1 = 1600 \text{ Hz; } f_2 = 1800 \text{ Hz; } f_3 = 2000 \text{ Hz; } f_4 = 2200 \text{ Hz; } f_5 = 2400 \text{ Hz and } f_6 = 2600 \text{ Hz} \]

which allows you to hear a harmonic tone in the sound fragment:

[Sound E 7 0]

**Experiment 3.7.1**

To the 7-tone complex in Experiment 3.7.0 we have added the twin-tone contribution:

\[ f_7 = 201 \text{ Hz; } f_8 = 402 \text{ Hz; with an Extra Multiply Amplitude of } \sqrt{6} = 2.449 \]

to create the 9-tone complex:

\[ f_0 = 1400 \text{ Hz; } f_1 = 1600 \text{ Hz; } f_2 = 1800 \text{ Hz; } f_3 = 2000 \text{ Hz; } f_4 = 2200 \text{ Hz; } f_5 = 2400 \text{ Hz; } f_6 = 2600 \text{ Hz; } f_7 = 201 \text{ Hz and } f_8 = 402 \text{ Hz} \]

you can hear a harmonic tone with a lower timbre than in [Sound E 7 0], however with a 1 Hz beat in the sound fragment:

[Sound E 7 1]

**Experiment 3.7.2**

To the 7-tone complex in Experiment 3.7.0 we have added the twin-tone contribution:

\[ f_7 = 199 \text{ Hz; } f_8 = 398 \text{ Hz; with an Extra Multiply Amplitude of } \sqrt{6} = 2.449 \]

you will hear an almost identical harmonic tone as in Experiment 3.7.1, again with a 1 Hz beat in the sound fragment:

[Sound E 7 2]
Experiment 3.7.3

We have now filled in the frequencies:

\[ f_0 = 1430 \text{ Hz}; f_1 = 1630 \text{ Hz}; f_2 = 1830 \text{ Hz}; f_3 = 2030 \text{ Hz}; f_4 = 2230 \text{ Hz}; \]
\[ f_5 = 2430 \text{ Hz} \text{ and } f_6 = 2630 \text{ Hz} \]
you will be able to hear an enharmonic tone in the sound fragment:

[Sound E 7 3]

Remark

Listeners who are trained in music are able to distinguish the pitch and its shift upward, as reported by De Boer and others.
In our experience the sound is rather harsh.

Experiment 3.7.4

In his thesis [A I 1] De Boer has explained that the pitch of Experiment 3.7.3 can best be compared to the pitch of the harmonic tone complex for which we have filled in:

\[ f_0 = 1421 \text{ Hz}; f_1 = 1624 \text{ Hz}; f_2 = 1827 \text{ Hz}; f_3 = 2030 \text{ Hz}; f_4 = 2233 \text{ Hz}; \]
\[ f_5 = 2436 \text{ Hz} \text{ and } f_6 = 2639 \text{ Hz} \]
which has a 203 Hz fundamental frequency.

And with that frequency series you can hear a harmonic tone in the sound fragment:

[Sound E 7 4]

Remark

Here, according to previous results the pitch is equal to 203 Hz, the first harmonic of this series of 7 successive higher harmonics.

Experiment 3.7.5

To the 7-tone complex in Experiment 3.7.4 we have added the twin-tone contribution:

\[ f_7 = 203 \text{ Hz}; f_8 = 406 \text{ Hz}; \] with an Extra Multiply Amplitude of \( \sqrt{6} = 2.449 \)
to create the 9-tone complex:

\[ f_0 = 1421 \text{ Hz}; f_1 = 1624 \text{ Hz}; f_2 = 1827 \text{ Hz}; f_3 = 2030 \text{ Hz}; f_4 = 2233 \text{ Hz}; \]
\[ f_5 = 2436 \text{ Hz}; f_6 = 2639 \text{ Hz}; f_7 = 203 \text{ Hz} \text{ and } f_8 = 406 \text{ Hz} \]
you will hear a harmonic tone with a lower timbre than in Experiment 3.7.4, without a beat, in the sound fragment:

[Sound E 7 5]
Remark

Therefore, we made the attempt to search for this pitch in the enharmonic 7-tone complex of Experiment 3.7.3, by means of the pitch tracing mechanism. This mechanism consists of the addition of several different twin-tones to this tone complex, and to subsequently observe the changes in evoked beats.

In Experiment 3.7.6 we have started with the twin-tone of 203 Hz and 406 Hz because we can expect the pitch of 203 Hz De Boer has predicted. And if he is correct you will hear no beat with this twin-tone test.

Experiment 3.7.6

To the 7-tone complex in Experiment 3.7.3 we have added the twin-tone contribution:

\[ f_7 = 203 \text{ Hz}; f_8 = 406 \text{ Hz}; \text{with an Extra Multiply Amplitude of } \sqrt{6} = 2.449 \]

to create the 9-tone complex:

\[ f_0 = 1430 \text{ Hz}; f_1 = 1630 \text{ Hz}; f_2 = 1830 \text{ Hz}; f_3 = 2030 \text{ Hz}; f_4 = 2230 \text{ Hz}; f_5 = 2430 \text{ Hz}; f_6 = 2630 \text{ Hz}; f_7 = 203 \text{ Hz and } f_8 = 406 \text{ Hz} \]

you will hear an enharmonic tone with a lower timbre than in Experiment 3.7.3. However, instead of hearing an extra intensity to the pitch as expected, you will clearly hear a 3 Hz beat in the sound fragment:

[Sound E 7 6]

Experiment 3.7.7

We have replaced the added twin-tone in Experiment 3.7.6 by the twin-tone:

\[ f_7 = 202 \text{ Hz and } f_8 = 404 \text{ Hz} \]

you will hear a beat of 2 Hz in the sound fragment:

[Sound E 7 7]

Experiment 3.7.8

We have replaced the formerly added twin-tone by:

\[ f_7 = 201 \text{ Hz}; f_8 = 402 \text{ Hz} \]

you will hear a beat of 1 Hz in the sound fragment:

[Sound E 7 8]
Experiment 3.7.9
We have replaced the formerly added twin-tone by:
\[ f_7 = 200 \text{ Hz}; f_8 = 400 \text{ Hz} \]
you hear that the beat completely disappears in the sound fragment:
[Sound E 7 9]

Experiment 3.7.10
We finally have replaced the formerly added twin-tone by:
\[ f_7 = 199 \text{ Hz and } f_8 = 398 \text{ Hz} \]
you will hear that the beat returns to 1 Hz in the sound fragment:
[Sound E 7 r]

Remark

As no examiner can find any trace of a shift in pitch in these beat experiments, the only possible conclusion is that the pitch shift, which is observed and reported by trained listeners in music perception, actually is an illusion.

The actual pitch remains unchanged at 200 Hz, which is the smallest difference frequency in the enharmonic tone series.

3.8. Infrasound can also be heard according to the squaring principle

Experiment 3.8.0
When we fill in the twin-tone frequencies:
\[ f_0 = 600 \text{ Hz}; f_1 = 610 \text{ Hz} \]
you presume you hear a tone of 605 Hz with a vibrato of 10 Hz in the sound fragment:
[Sound E 8 0]

Remark

This is commonly explained as the beat effect of the combination of the two nearby frequencies. However that is not how it works, as can be seen in the following series of experiments.
Experiment 3.8.1

We have filled in the triplet frequencies:

\[ f_0 = 600 \text{ Hz}; f_1 = 610 \text{ Hz} \text{ and } f_2 = 620 \text{ Hz} \]

you will hear a tone of 610 Hz with a beat in the form of a vibrato of 10 Hz in the sound fragment:

[Sound E 8 1]

Experiments 3.8.2 – 3.8.3

By changing the \( f_1 \) in either 609 Hz or 611 Hz we have created the triplet:

\[ f_0 = 600 \text{ Hz}; f_1 = 609 \text{ Hz} \text{ (or } f_1 = 611 \text{ Hz}) \text{ and } f_2 = 620 \text{ Hz} \]

you will hear a tone of 610 Hz with a 10 Hz vibrato, and an extra beat of 2 Hz in the sound fragment:

[Sound E 8 2] respectively [Sound E 8 3]

Experiments 3.8.4 – 3.8.5 – 3.8.6

We have added one of the following combinations to the triplet of Experiment 3.8.2 or 3.8.3:

- \( f_3 = 580 \text{ Hz} \text{ and } f_4 = 590 \text{ Hz} \)
- or: \( f_3 = 590 \text{ Hz} \text{ and } f_4 = 630 \text{ Hz} \)
- or: \( f_3 = 630 \text{ Hz} \text{ and } f_4 = 640 \text{ Hz} \)

you will hear that the extra 2 Hz beat is changed into a 1 Hz beat in the sound fragments:

[Sound E 8 4]; [Sound E 8 5] respectively [Sound E 8 6]

Remark

You have to consider that the only frequency that can be modulated is the 10 Hz pitch.

Experiment 3.8.7

To the triplet in Experiment 3.8.2 or 3.8.3 we have added another triplet:

\[ f_3 = 700 \text{ Hz}; f_4 = 710 \text{ Hz} \text{ and } f_5 = 720 \text{ Hz} \]

you will hear that, apart from a change in timbre, the extra 2 Hz beat is changed into a 1 Hz beat in the sound fragment:

[Sound E 8 7]
Remark

The combination of the four difference frequency contributions of 9; 2 times 10 and 11 Hz results in a signal of 10 Hz with a beat of 1 Hz. Which is halved compared to the beat in Experiment 3.8.2 or 3.8.3.

Experiment 3.8.8

In the added triplet in Experiment 3.8.7 we changed the frequency $f_4 = 710$ Hz into $f_4 = 711$ Hz and thus this triplet became:

$f_3 = 700$ Hz; $f_4 = 711$ Hz and $f_5 = 720$

you will hear that the extra 1 Hz beat again changes into a 2 Hz beat in the sound fragment:

[Sound E 8 8]

Experiment 3.8.9

When we change the phase of the $f_4$ into 180° in Experiment 3.8.8 you will hear that the 10 Hz beat disappears almost completely. Instead of that you hear a not modulated 20 Hz vibrato in the sound fragment:

[Sound E 8 9]

Remark

This occurs because the two difference frequencies of 9 Hz and 11 Hz (evoked out of the 600 + 609 (or 611) + 620 Hz triplet) and the same two difference frequencies of 9 Hz and 11 Hz (evoked out of the 700 + 709 (or 711) + 720 Hz triplet) are – pair by pair – practically equal in amplitude and have an opposite phase. Therefore they cancel each other’s contributions almost completely.

What remains is the combination of two contributions of the difference frequency of 20 Hz as a result from the combinations 620 – 600 and 720 – 700 Hz, heard as a 20 Hz vibrato.

References
